1996

Allocation of Two Dimensional Parts Using a Shape Reasoning Heuristic.

Henry Joseph Lamousin III
Louisiana State University and Agricultural & Mechanical College

ALLOCATION OF TWO DIMENSIONAL PARTS USING A SHAPE REASONING HEURISTIC

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Mechanical Engineering

by
Henry J. Lamousin III
B.S., University of New Orleans, 1987
December 1996

UMI Number: 9720365

Acknowledgments

First and foremost I would like to thank my mother, Rosemary, for her patronage and patience throughout what by all measures has been a lengthy graduate career. Without her generous support much of the work accomplished would not be have been possible. I would also like to acknowledge Mr. Jules Hubert for the companionship and wise council he has offered my mother throughout my studies. His friendship has proven invaluable in achieving my goals.

I would like to express appreciation to the professors serving on my committee for their time and efforts. These are Doctors Sumanta Acharya, T. Warren Liao, Michael Murphy, Dirk Smith, Neal Stoltzfus, and Warren Waggenspack. In particular I would like to thank Doctors Smith and Murphy for the many helpful suggestions they have made in editing this manuscript. I am also grateful to Dr. Waggenspack for his service as my major professor, and for the liberties and professional courtesies granted me in conducting this research.

I would like to thank McDermott Inc. for the stimulus and original funding behind this work, and recognize the efforts of Mr. Mark Adams in contributing the data sets used for evaluation. This work was also supported in part by a fellowship from the Board of Regents.

A pledge of gratitude is accorded the many individuals who have tolerated my pranks, extreme moods, and endless diatribes over the years. My fellow lab mates have taken the brunt of this burden, including: Robert Benton, Sandeep Danni, Siddarameshwar Bagali, Yohannes Desta, Olalekan Odesanya, David Thompson, Ramji Venkatachari and Tarek Zohdi. I would also like to express my gratefulness to Aleric Haag for his endeavors in maintaining the computer facilities of the Interactive Modeling Research Laboratory, where the majority of this work was completed. To all of you and numerous others, thanks.

Finally, I would like to convey my deepest appreciation to two special individuals, Laura Gough and Gregory Dobson. Laura has shared in the emotional highs and lows, provided a much needed calming voice on many occasions, and proved an indispensable companion over these last two trying years. The value of Greg's service as a sounding board for the numerous solutions to my assorted technical and personal problems cannot be calculated. I owe my success and sanity in great part to his good advice and unwavering friendship over the years. Thanks!

Table of Contents

List of Tables

List of Figures

Abstract

A technique is outlined for the allocation of irregular parts onto arbitrarily shaped resources. Placements are generated by matching complementary shapes between the unplaced parts and the remaining areas of the stock material. The part and resource profiles are characterized to varying levels of detail using geometric "features". Information contained in the features is used at each stage of processing to intelligently select and place parts on the resource. Techniques for the efficient handling of complex profiles and other practical implementation issues are described. The utility of the proposed approach is verified using diverse problems from a marine fabrication facility. The formulation and performance of the method is contrasted to previously published works.

Chapter 1
Introduction and Literature Review

1.1 The Allocation Problem in Industry

The task of allocating a set of smaller objects from a larger resource is common to many industries. Commonly referred to as *cutting and packing problems*, generic one, two and three dimensional examples are illustrated in Figure 1.1. A representative one dimensional case is found in the lumber industry where several standard stock lengths must be cut from trees of varying sizes. A similar problem exists in paper production where orders for different roll widths must be cut from standard machine spools. Two dimensional applications come from the furniture, canvas, and glass industries where rectangular pieces are allocated from larger stock materials. The more complicated task of laying out irregular (non-rectangular) shapes is required for garment production in the textile trade. This is also required for the fabrication of ships, offshore platforms and most other products produced from sheet metal materials. Cargo container packing and pallet loading are the characteristic of the three dimensional case.

Two immediate goals are evident from the automation of these tasks, the reduction of waste and the number of man hours required for the layout problem. Often a mix of automatic and manual methods is used. Automation improves material requirement estimates and reduces the time required for initial part layout. Final layouts can many times be improved with manual alteration; however, any gains made through interaction must be weighed against the increased cost in man hours. In practice the problem requires optimization of both material and labor used.

Several other equally important criteria must also be considered. These include, but are not limited to: scheduling constraints imposed by stock material inventories, production scheduling, and the availability of fabrication machinery [SPER79]. Manufacturing equipment can impose other geometric limitations on the layout, such as requiring guillotine cuts or additional spacing between profiles to account for the curf of flame cutting devices. Even with these restrictions, automation when implemented properly can help reduce overall production costs.

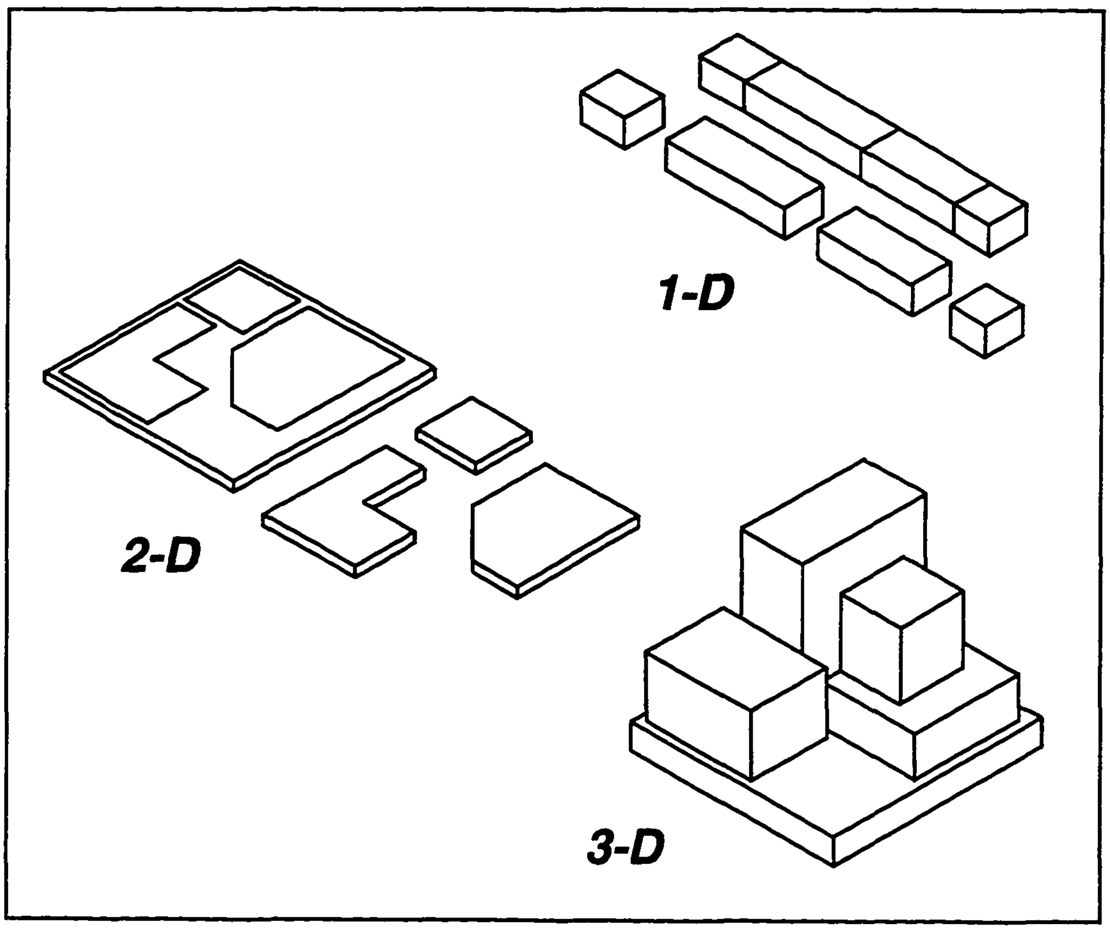


Figure 1.1 Three examples demonstrating the one, two and three dimension aspects of the allocation problem.

The work presented in this dissertation deals with a general case of the two dimensional *allocation* problem, often called *nesting*. Although capable of dealing with less complex forms, the focus of interest was on solving problems involving complex shapes. The objective was to generate layouts automatically, based on reasoning about the part and stock shapes involved. The technique developed is conceptually similar to the manual method used in solving jigsaw puzzles.

Before detailing the actual focus of this project, a brief overview is provided in the next section, of the various attributes used to characterize and classify the numerous problems found in industry. The particular class of problem studied in this work is then described and a brief history of the project given. The remainder of the chapter is devoted to reporting on previous attempts at solving this problem in the literature. Finally, the proposed technique is discussed in light of this previous research, and an outline of the remainder of the dissertation is given.

1.2 Classification of Allocation Problems

Numerous techniques for automating cutting and packing problems can be found in the literature. Much of this research is summarized in the surveys of Haessler, Sarin, and Hinxman [HAES91] [SARI83] [HINX80]. However, Dychhoff goes one step further and proposes a typology of all allocations problems based upon their fundamental logical structure [DYCK90]. The purpose of the typology is to provide a consistent and systematic approach for condensing the myriad of problems investigated, and to unify the different notions used throughout the literature. This concept was expanded upon in the book *Cutting and Packing in Production and Distribution, A Typology and Bibliography*, where four general problem types were defined [DYCK92]. Since the goal of the book was to serve as an aid in selecting and developing solution procedures, these types have similarities in solution approaches. The problems associated with each type are described using entries from a catalogue of problem characteristics or attributes. The object in defining the four types was not to use all attributes, but only those which are significant in terms of solution procedures. Some of these "type-defining" characteristics are summarized next.

Two basic data types exist for all allocation problems, the *stock*, or *plates*, or *resources*, and the smaller parts which are packed into or cut from them. These may also be referred to as *items*, *shapes*, or *profiles*. Although the individual properties for each data type differ for a given problem, they may be described using the same characteristics. The first of characteristic, dimension (1-D, 2-D, 3-D), has already been introduced.

Next is *shape*, where in 2-D problems a distinction is made between rectangular and irregular (non-rectangular) profiles. An analogous division between orthogonal parallelopipeds (boxes) and all other volumes can be made for the 3-D case. Although less common, further classifications such as convex, non-convex, triangular, four sided, etc., can also be made.

Assortment and *availability* are the remaining characteristics used to describe the two data types. Assortments may be either homogeneous or heterogeneous. The heterogeneous case may be further classified based on the existence of duplicated shapes, which permit division into representative groups. For resources, availability refers to the number of stocks accessible during the solution process, while for parts availability defines the upper limit on items used or produced. Cases with infinite stocks and/or parts occur frequently in industry.

Cutting and packing problems may also be characterized based on the nature of the assignment involved. Dyckhoff enumerates four types:

Type I: All resources and parts used,

Type II: All resources used, a selection of parts used,

Type III: All parts used, a selection of resources,

Type IV: A selection of parts and resources.

Type I is a pure layout problem as when a set of machinery must be distributed over a factory floor. For type II assignments, a selection of the available parts is required to efficiently use all resources. Conversely, for Type III all parts must be allocated with emphasis often placed on selecting the minimum resources required. Type IV is a mix of both II and III.

The objective in carrying out assignments is also an attribute describing the problem. The key goal is always focused on reducing waste or maximizing profits; however, the exact criteria differ from problem to problem. Less obvious issues such as lowering inventory storage and handling costs, and minimizing change-over and cutting times may also come into play.

Geometric restrictions placed on the arrangement of parts constitute yet another factor describing the allocation problem. For the majority of cases, parts may not overlap and must fall totally within the resource. Restrictions upon the orientation of profiles can exist due to directional properties of the stock, as is the case for wood, milled steel, and many composite materials. The fabrication equipment may also place limitations on the patterns used. Arrangements where part boundaries are parallel to stock boundaries (orthogonal patterns) are often required for paper cutting devices. The ability to perform straight uninterrupted cuts from one end of the resource to the other (*guillotine cuts*) is commonly seen in this industry (Figure 1.2). Often, when processing sheet metal, spacing between parts must be allowed to account for the curf of the cutting device. Defects in the material itself can also affect the layout patterns.

Dyckhoff also distinguishes four basic problem types, distinct from the four assignment types mentioned earlier, based on the

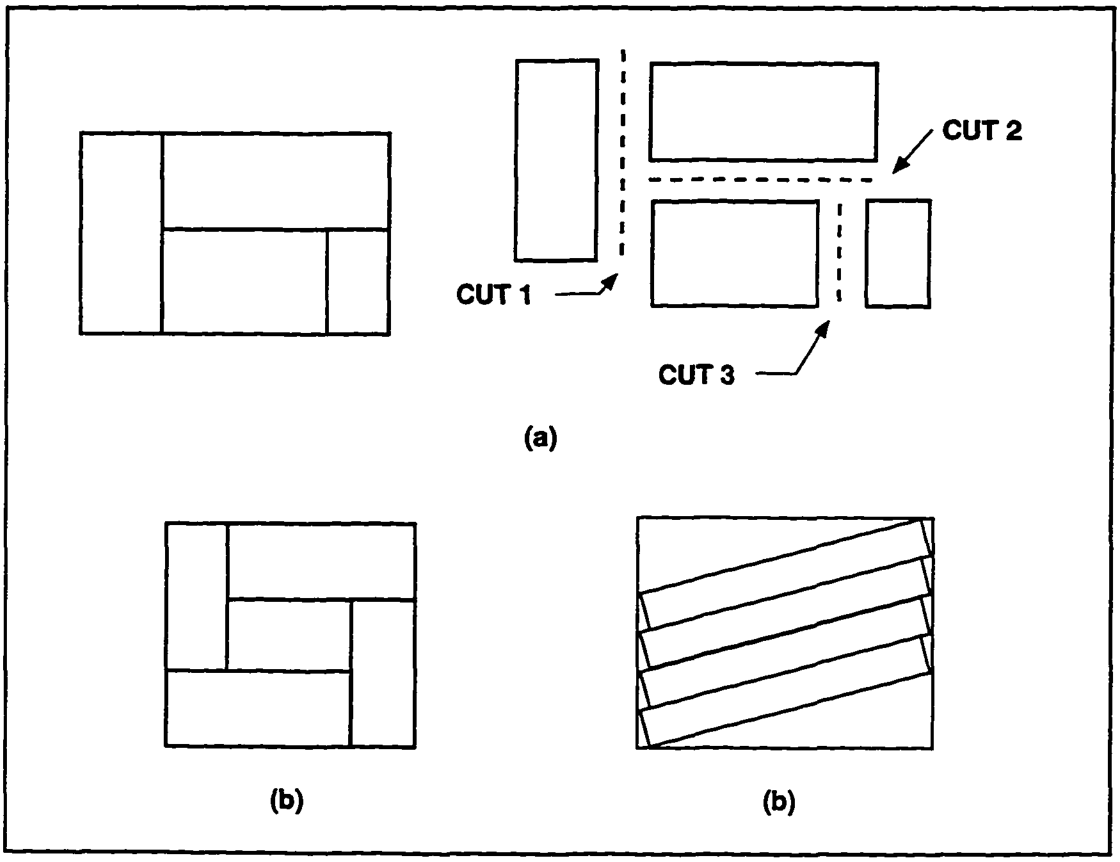


Figure 1.2 Different layout or pattern types: (a) guillotine, (b) non-guillotine or nested, (c) non-orthogonal.

above criteria: cutting stock, bin packing, knapsack, and pallet loading. *Cutting stock* problems consist of large heterogeneous assortments of parts with only a few distinct shapes which must be allocated to a selection of stocks either completely or incompletely. Since the items may be divided into a groups of identical shapes, repetitive solutions can often be used for each resource. By contrast, the *bin packing* type also contains large heterogeneous assortments of parts, but multiple identical parts are not common. Consequently, this type of problem is more complex, as assignments for each part and solutions for each resource must be considered individually. In *knapsack* problems, a large, often infinite supply of heterogeneous patterns must be assigned to a limited set of stock. For this type of problem, all resources must be used to complete the solution. Finally, the *pallet loading* type consists of problems where a selection of homogeneous parts must be assigned to a generally homogeneous set of stocks.

1.3 The Problem Investigated

The problem investigated represents a special case of the two dimensional bin packing type often called nesting. The assortment of parts is variable in nature. Profiles vary from rectangles to highly irregular and complex shapes involving chamfers, fillets and general curved edges. The distribution of profiles is not characterized by multiple identical copies and thus effectively eliminates the repetitive use of layouts as a solution. There is no orientation restriction on the placement of parts. The problem is further generalized by the requirement that, where possible, items allocated be placed within the irregular holes or void regions existing within some larger part profiles. All other resources or stock materials specified by the user are rectangular. Various sizes and grades are permitted, however the task of selecting the optimum stock is not addressed [GEMM92] [QU89].

Research for this project was initiated by a request from a ship and offshore oil platform fabricator seeking to automate material requirement estimates for large projects. Proprietary systems are available from CAMSCO and Gerbers Scientific for ship building while Microdynamcis, Assyst Inc., Inverstronica and others offer products for the textile industry. None of the commercial packages satisfied the particular requirements of this project. Furthermore, such algorithms are proprietary and could not easily be incorporated into the sponsor's current operations. A one year pilot study was intiated and algorithms were developed to address the problems of interest. The results of this initial study are found in Chapter two. Following this, an independent investigation was conducted to develop a new, more innovative, and efficient solution to the problem. Although the majority of the research presented was conducted during this phase, many of the strategies introduced were motivated by or grew out of the results of the pilot study. Solutions from the initial study were also used as a basis of comparison for the new technique.

1.4 Classification of Solution Techniques

Irregular profile bin packing falls into a class of problems which are NP complete [GARE79]. An infinite number of solutions are possible and it is seldom possible to determine a true best solution for any given set of patterns. In the absence of an analytically defined optimum, a suitable goal for automation is a set of solutions equal in quality to those of manually produced layouts.

Solution strategies for this type of problem can be separated into two broad tasks, part selection and part placement. *Part placement* focuses on identifying locations and orientations for profiles that satisfy the geometric restraints imposed on the solution. This is often referred to as *layout* or (cutting) *pattern generation*. *Part selection* or *assignment* deals with choosing the best parts for each stock plate in the solution.

An interesting and familiar analogy can be found in the game of chess. Here the generation of part placements is a relatively simple task. A limited set of permissible moves for each piece are explicitly defined by a set of easily evaluated rules. For example, bishops may only move any unobstructed distance along a diagonal, while rooks are restricted to purely horizontal or vertical movements. For the layout problem, an infinite number of locations and orientations are available for each of many parts, while the non-overlapping constraint makes validating a placement substantially more complex. As with chess, selecting the best move or placement is very difficult. Often parts are selected and placed in an iterative fashion as with chess moves. For such an approach it is often difficult to predict with certainty the future effect which any placement may have on achieving the overall objective of the problem as the number of possibilities is infinite. While preventing the capture of the king in chess is difficult, optimal packing of highly irregular shapes is an equally daunting task. Nor can the tasks be considered independent. The interrelation between the layout and assignment is critical, as optimal selection of parts is rendered meaningless without proper placement.

A review of the literature shows limited research into this problem. In Dyckhoff's bibliography only 23 of the 142 two dimensional problems cited involve irregular shapes. The majority of these are cutting stock types. Due to the inherent complexity of nesting, almost all solutions employ some form of heuristic. The previous research can most easily be classified based upon the method in which part placements are generated. Using this method, three broad categories can be defined: 1) rectangular approximation strategies; 2) optimization methods; and 3) rule based, intelligent or expert system approaches. Each group is detailed further in the following sections.

1.4.1 Rectangular Approximation Placement Strategies

These methods reduce the complexity of handling irregular profiles by approximating either individual or groups of parts with rectangular enclosures. With all pieces resolved to this form, the algorithms for nesting rectangular shapes are directly applicable. As previously mentioned, substantial research has been conducted in this area [HAES91].

Much of the earliest work on the rectangular two dimensional allocation problem is credited to Gilmore and Gomory, who showed that it could be formulated as a linear programming problem [GILM65]. Unlike the one dimensional case, the two dimensional problem could not be solved using a knapsack function, as efficient solutions to higher order knapsack problems were not available. However, with certain restrictions, such as guillotine cuts, algorithms for optimal and near optimal solutions could be formulated. Since then other techniques have been used, including: recursive algorithms, dynamic programming, combinatorics, and various heuristics [ISRA82]. A representative set of these methods may be found in references by Adamowicz, Bengtsson, Christofides, Dagli, Dietrich, Farley, Hahn, Nee, and Oliveira [ADAM76], [BENG82], [CHRI77], [DAGL88], [DIET91], [FARL90], [HAHN68], [NEE88], [OLIV90].

In their simplest form, rectangular approximation methods replace each part profile by its *minimum enclosing rectangle* (MER) or smallest bounding box. Efficient methods for generating a polygon's MER are detailed by Freeman [FREE75a] and Adamowicz [ADAM79]. The quality of the nests produced is dependent to a large extent on the accuracy of the MER; as all waste associated with this approximation is automatically included in the solution. For highly irregular shapes this waste can be significant (Figure 1.3). In order to partially overcome this limitation, some have proposed clustering pieces together to form rectangular modules which are subsequently nested [ADAM79], [NEE86], [HAIM70]. This reformulates the problem as a series of smaller layouts, since generation of each module is, in essence, the placement of pieces onto a smaller resource. To be effective, this technique requires parts which readily combine to form rectangles. Small parts are often required to fill the waste areas encompassed by the rectangular enclosure. For many applications these conditions cannot be met. Furthermore, parts which are large relative to the available resource can not be clustered.

1.4.2 Optimization Methods

The second group of approaches proposed in the literature make use of several optimization techniques for minimizing trim waste, including *multi-start non-gradient searches*, *neural networks* [POSH90], [CAVI89], *simulated annealing* [DAGL90a], [DAGI90b], [KAMP88], and *genetic algorithms*. Although implementation issues differ with

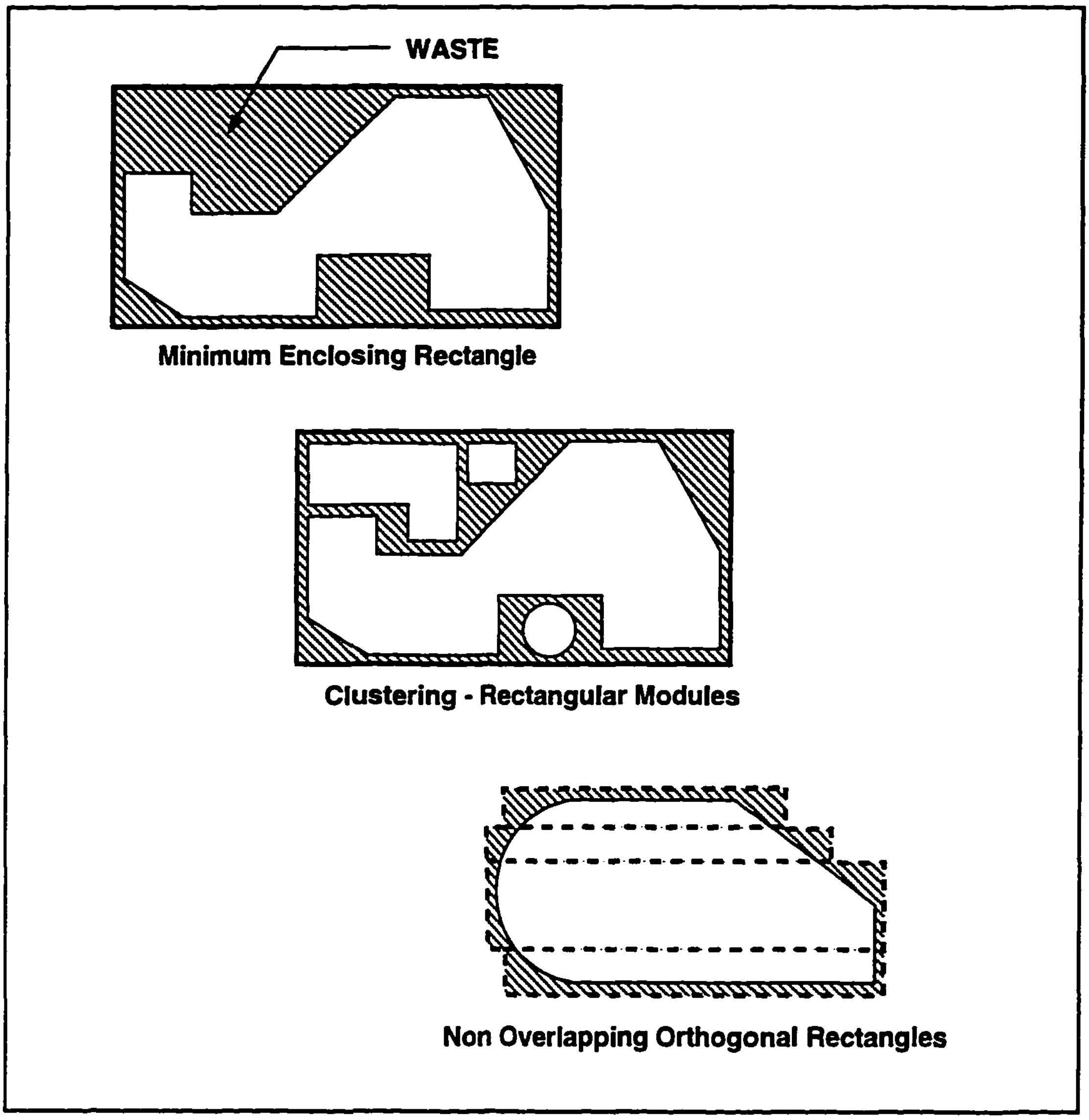


Figure 1.3 Rectangular Approximation Methods

individual techniques, there are two common requirements for all. First, an objective function must be formulated in terms of the input parameters and variables of the problems. In the most general case, this represents three unknowns for each part, corresponding to its location (x,y) and orientation (θ). The second requirement is a technique for altering the current solution to generate new ones. Unlike the other strategies presented in this review, optimization techniques do not generate solutions through the iterative placement of individual parts onto the resource. Rather, some initial solution including all parts is improved through various perturbation methods.

1.4.2.1 The Objective or Cost Function

The objective function may be formulated in several ways. However, when minimized it must generate solutions conforming to the geometric constraints imposed on the system. Since most applications require non overlapping placements, one term representing the area of intersection between parts can be found in almost all cost functions. Determining this term constitutes the majority of solution time, thus various methods for its efficient calculation have been proposed. If only an approximate measure is desired, a quadtree approach, as presented by Sandgren, may be used to achieve any desired level of accuracy [SAND88]. However, computation expense increases as the minimum grid size used to describe profiles decreases. A more analytical approach for finding the exact area of intersection is demonstrated by Jain [JAIN92]. Results from computational geometry show that calculation of the overlap between two polygons with N sides requires on the order of N^2 time [PREP85]. Consequently, as the number and complexity of parts increases, the expense of calculating this term can become substantial.

Beyond preventing overlap, a term controlling the overall size of the layout must be present in the objective function to reduce the resulting trim waste. If the dimensions of the used resource are unconstrained, the area of the rectangle encompassing all patterns is minimized. A useful alternative is to minimize the sum of the distances between each profile and the origin. For most other cases where the stock material is of a set width, the appropriate restrictions are applied to the layout and overall length is controlled. The following metric, D, may also be minimized

$$D = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left(w_{ij} \, d_{ij} \right) \qquad [1.1]$$

where

d_{ij} = the distance between parts i and j,

w_{ij} = the affinity or attraction of part i for part j.

Various bases for calculating the attraction of one part for another may be measured such as maximum edge-wise adjacency or the area of the smallest enclosing rectangle for the various part pairs. The desired goal of such terms is to help drive the optimization method toward effective part placements and layout solutions.

One difficulty in solving the allocation problem using an optimization formulation is the potential for numerous local minima in the cost function. Two strategies exist to overcome this problem. In multistart methods, the random selection of many starting points allows for the location of the different local minima. The smallest result is chosen as the global minima. If this technique is used, traditional "downhill" search algorithms can be used to improve the layout. However, due to the non-differentiable nature of the objective function, non-gradient based techniques are generally required. The second approach is to choose a search capable of "jumping out of" the valleys associated with local minima, such as simulated annealing or genetic algorithms.

1.4.2.2 Simulated Annealing Techniques

Simulated annealing is a "probabilistic hill climbing optimization technique" [JAIN92] [KIRK83]. The basic steps for implementing a simulated annealing algorithm are shown in Figure 1.4. The overall structure consists of two loops. Within the inner loop, the current configuration or layout k_i is altered in a random fashion to produce another configuration k_j within its neighborhood. The layout generated may be accepted based on one of two criteria. If the cost of the objective function for k_j is less than or equal to that of k_i it is accepted. Otherwise, the acceptance of k_j is based upon the generation of a random number between zero and one. If this number falls below:

$$e^{-\Delta C/T} \qquad [1.2]$$

where

$$\Delta C = C(k_j) - C(k_i) \qquad [1.3]$$

and $C(\,)$ represents the value of a configuration's cost function, k_j is accepted and becomes the current configuration. Otherwise the original configuration is retained. Processing within the inner loop continues until equilibrium is reached, which for nesting applications is usually defined as the generation of a preset number of accepted configurations. The algorithm then jumps to the outer loop where the controlling parameter or temperature, T, is decremented. Processing continues in this way until the temperature is small enough to prevent any substantial improvement to the solution. The key advantage of the method is its ability to accept intermediate solutions with higher cost values in a controlled way.

Various techniques have been used to generate the necessary intermediate layouts, which are often referred to as *moves*. One method is to interchange the position of two objects [LUTF92]. More typical, however, is the random perturbation of a part's position (x,y) and orientation (θ). Since the new configurations must fall within the

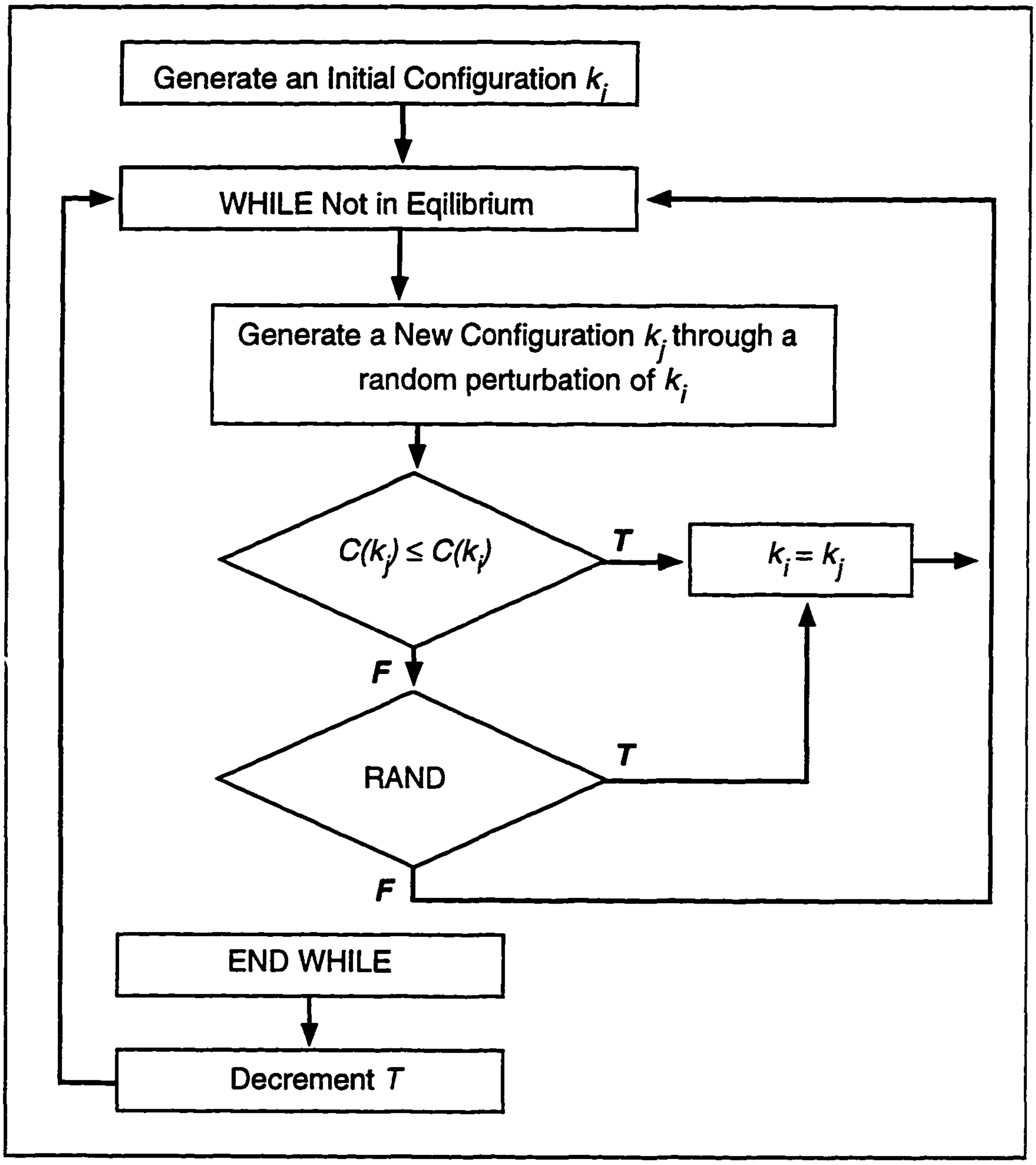


Figure 1.4 Flow diagram showing the basic steps of simulated annealing

neighborhood of the old configurations, the magnitude of change is usually limited. In most implementations, as the temperature is lowered, the extent of part movement is reduced, causing a corresponding reduction in the size of the associated neighborhood.

A cooling schedule specifies the way in which the temperature is decremented and determines the criteria for equilibrium at each stage. Proper selection of the cooling schedule is critical. If the initial temperature is set too low or cooling proceeds too fast, the solution may be trapped in a local minima. If the temperature is set too high and decrements too small, the method will progress unnecessarily slowly. Selection of the cooling schedule is usually done by trial and error based on information gained from prior runs with a similar structure.

1.4.2.3 Genetic Algorithms

Another probabilistic approach to solving the nesting problem is to combine *genetic algorithms* with a local minimization routine [FUJI93]. In order to handle solutions in a manner analogous to genes, layouts must be formulated as strings. For this purpose, solutions are represented as an ordered list of parts (e.g. part A, part B, . . ., part J). The location of each part in the layout is measured in the local coordinate system of the pattern immediately proceeding it in the list. A set number of such solutions is generated to start the process. At each stage of the algorithm, a *fitness* is assign to each member of this *population* of layouts. This fitness number corresponds to the value of the cost or objective functions mentioned previously. At each stage of the solution, a new *generation* of layouts is produced from those members having the highest fitness values, using three of *genetic operators*.

The three genetic operators used represent techniques for generating new solutions from the current generation of layouts or *idividuals*. The first and most frequently used operator is *ordered crossover*. A pair of individuals labeled parent 1 and parent 2 are selected and a crossover location in the ordered lists, representing their solutions, randomly selected. A *child* or new solution is produced in the following fashion. The portion of parent 1 lying to the left of the crossover point is copied directly into the child. The remaining profiles are copied into the child in the order in which they appear in parent 2 (Figure 1.5). The relative coordinates (x,y,θ) of each part are retained from its parent string with one exception. The coordinates of the part immediately following the crossover location are generated randomly. A second child can also be produced by exchanging the roles of parent 1 and parent 2 and repeating the process. The second genetic operator used is *mutation.* Here a random piece is removed from the solution and re-inserted at a random location in the list (Figure 1.5). For the final technique, *elitist selection*, a child is produced by copying the solution with the highest fitness directly into the new generation. The probability with which the three operators are applied is preset in the algorithm.

The solutions produced in the above fashion are seldom acceptable layouts. Consequently, a local minimization must be performed on each of the children. Analogous to multistart methods, overlap and layout dimensions are minimized using a "Quasi-Newton" method. The square of the distances between adjacent parts in the solution lists is also minimized. This overall process is continued until a set number of generations has been produced. The member of the population with the highest fitness is selected as the final solution.

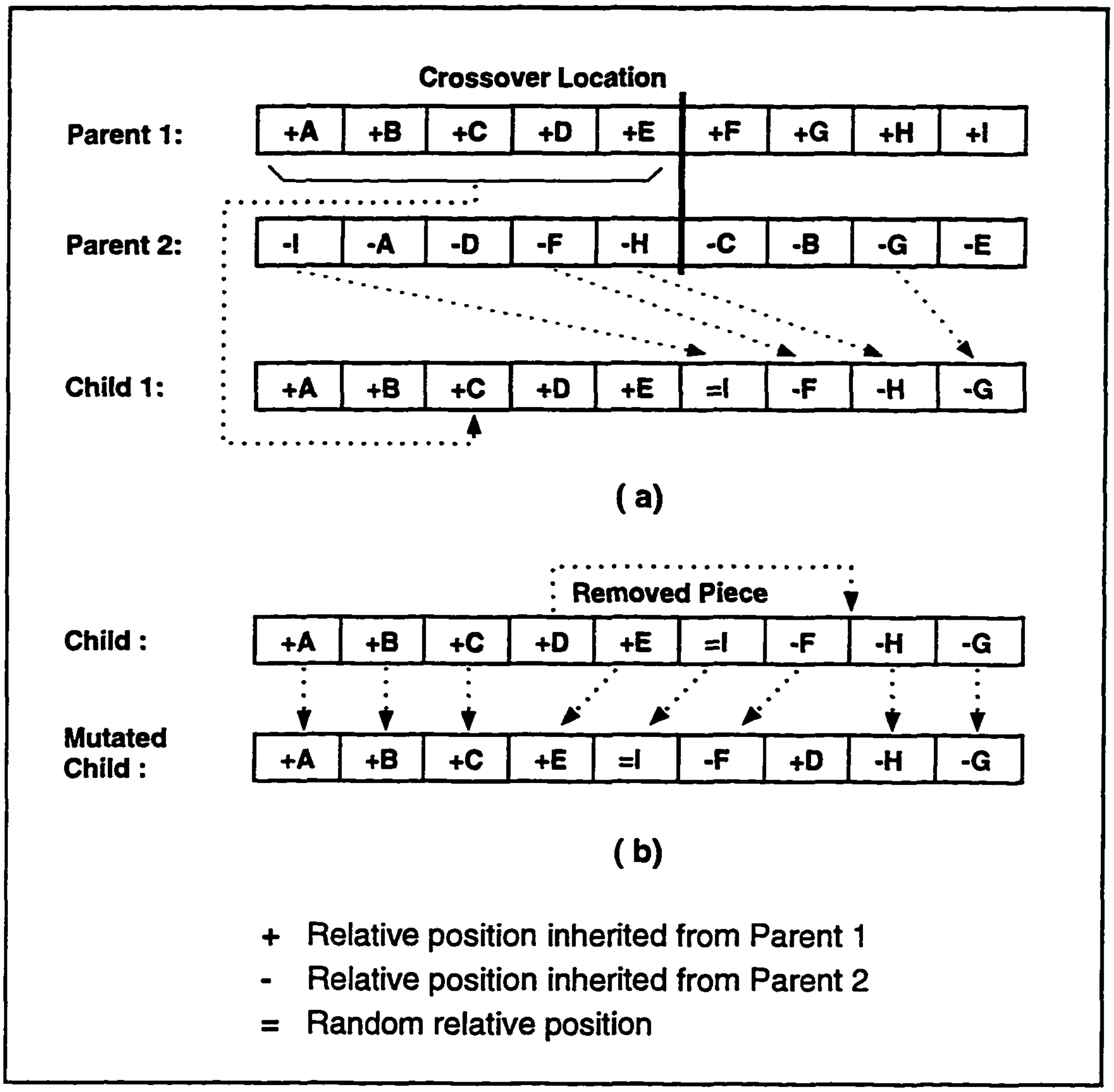


Figure 1.5 Operators used in the genetic algorithm: (a) Crossover, (b) Mutation

Although computationally expensive, optimization techniques offer the advantage of potentially finding a true minimum over the range of design variables. Unfortunately, the computation time associated with such solutions are often impractical. For example, the genetic algorithm referenced above required 12 hours to nest 12 parts. As a consequence, problem formulations are often limited to either relatively few parts or only convex patterns. Although such limitations are applicable to certain stamping operations, they cannot be reconciled to the problem of interest. Furthermore, no method for dealing with resources of limited dimension has been proposed.

1.4.3 Rule Based, Intelligent, & Expert Systems

The final group of solutions to the irregular 2-D bin packing problem are lumped into a category referred to as *rule based, intelligent* and *expert systems*. Although broad, all methods in this class differ from those solutions previously presented in two key ways. First, unlike rectangular approximation strategies, irregular geometries are used to represent the parts during placements, and second, solutions are generated by placing parts onto the resource one at a time in a sequential fashion. This differs from optimization strategies where a layout for all parts is initially present. Consequently, at each stage of the solution process, an infinite number of positions and orientations exist for those parts remaining to be placed. To reduce the possibilities to a reasonable number of choices, various heuristics are incorporated.

1.4.3.1 Fixed Orientation Boundary Abutting Techniques

Much of the variability in placement of parts can be reduced by restricting the number of different orientations allowed for profiles. No clear automated method for picking the optimum orientation exists, however efficient selection can often be done manually. With profiles confined in this way, satisfactory positioning can often be achieved by placing parts such that they touch but do not overlap.

Various methods for generating these placements are found in the literature. Freeman outlines a procedure for packing a single arbitrary shape multiple times; however, the profiles must first be approximated using a chain code [FREE75b]. An interesting hybrid of the rectangular approximation schemes is presented by Qu [QU87]. Each irregular profile is represented by a set of non-overlapping orthogonal rectangles (Figure 1.3). Adjacent and non-overlapping placements are easily determined since intersection of the component rectangles is a straight forward process. Packing proceeds by producing a series of stock width sized strips or layers which are then used to fill the resource from top to bottom. Although promising, this profile approximation technique restricts the final solution to orthogonal layout patterns. Furthermore, a part's representation is highly dependent on orientation. Even some relatively simple shapes can require a large number of rectangles for effective approximation, causing the approach to be computationally impractical.

The most general fixed orientation boundary abutting approach was presented by Albano and Sapuppo [ALBA80]. A geometric construct, referred to as the No Fit Polygon, is used to determine the locus of all points where a part may be placed such that it touches but does not overlap any already allocated part. Unlike other placement techniques, any polygonal representation of the parts can be used for calculations. By combining this method with a "placement policy" and a search heuristic, an effective method for solving the irregular bin packing problem was produced. A logical extension of this technique to more general problems was developed in the initial phase of this project. A more detailed discussion of the pilot study is deferred until Chapter 2, where the results are presented.

1.4.3.2 Shape Based Techniques

A key drawback of the techniques presented in the previous section is the limitation of the number of orientations which parts may take in order to reduce computational expense. Frequently only integer multiples of $\pi / 2$ are investigated (0°, 90°, 180°, 270°); however, even for rectangular shapes, these may not produce optimal nests [DECA78].

A more "intelligent" approach for selecting, orienting, and placing parts is to mimic one technique used by manual nesters. Here a search is conducted for complementary shapes among the unplaced part profiles, and the remaining usable stock profile. Solutions are generated by piecing together layouts in a puzzle like fashion. Reasoning of this type has been proposed to varying degrees by several authors.

In the most elementary form, shape reasoning represents the matching of profile sides. This is the approach taken by Dagli and Totoglu [DAGL87]. Patterns are allocated to plates sequentially, with the order determined by a set of priorities based on properties such as part area, profile perimeter, and complexity. Starting with the two highest priority parts, their relative locations are determined by pairwise matching each of their sides (Figure 1.6) and selecting the location yielding the smallest minimum enclosing rectangle (MER). The process is repeated with the next part profile in the prioritized list until all parts are placed, or no more room is available to place additional parts on the existing resource. The indiscriminate checking of all possible combinations of sides incurs the largest computational expense associated with the algorithm.

This basic principle of matching sides is used in another algorithm discussed by Prasad [PRAS94]. One part is held fixed while the other is slid or translated along its boundary as in the no fit polygon (NFP) described by Albano and Sappupo [ALBA80]. The orientation of the parts is determined by aligning the longest edges of their corresponding profiles. An MER is then constructed for each step of the NFP process. As with Dagli's method, the placement corresponding to the smallest MER is selected as best. Unfortunately, this algorithm is designed for

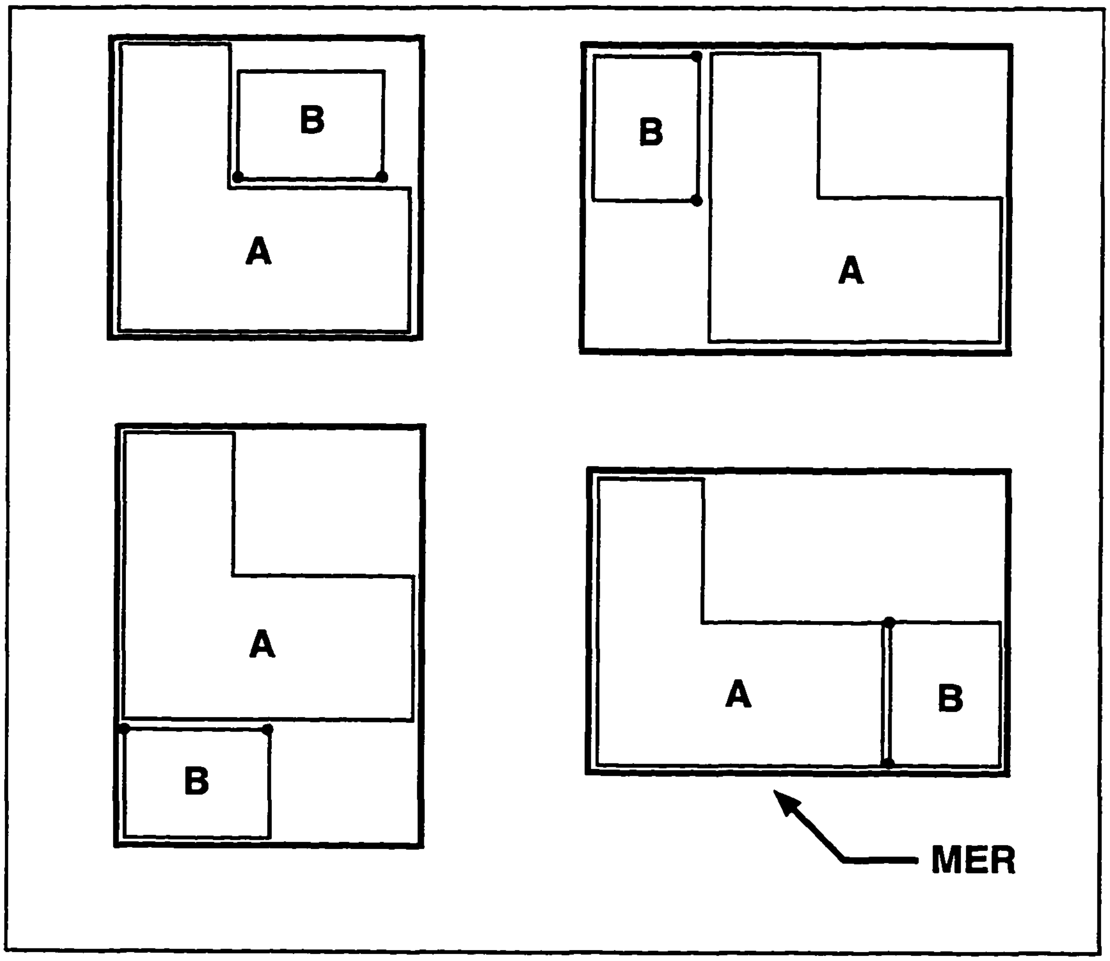


Figure 1.6 The method of Dagli and Totoglu. Part placements are generated by pairwise matches of all sides of parts A and B. The location producing the smallest MER is selected as best.

sheet metal stampings, and is limited to problems of only two or three parts.

Rule based or expert systems represent a second category of approach using shape to solve the nesting problem. One example, developed for a ship building application is presented by Cheok and Nee [CHEO91]. The automatic layout process is divided into three steps. In the first stage, called shape processing, an approximate description of the part profiles is produced by eliminating fillets, chamfers, and other minor features. These simplified shapes are then classified into groups based on commonly used parts such as floors, rectangular brackets, trapezoidal brackets, etc. (Figure 1.7). In the second stage, the classified parts are paired together according to predefined arrangements which, based on previous experience, produce "good" or tightly packed rectangular modules. These modules are nested in the third stage using a specialized rectangle packing method.

A second example of a rule based method is discussed by Yazu [YAZU87], where the shapes are based on clothes patterns. A large set of specific rules is used for each clothing type, such as men's shirts. Details concerning these rules and their use are not clearly presented in the discussion.

A key limitation of rule based techniques is their domain dependency. That is, they are very case specific. Consequently, the heuristics often break down in the context of general nesting, where unexpected situations occur, causing undesirable results.

A familiar and interesting analogy to the nesting problem, that of putting together jigsaw puzzles, was investigated by Freeman and later by Radack and Badler [FREE64], [RADA82]. Radack and Badler's study determined matches based on a novel method for representing the part profiles using a boundary-centered polar encoding. Freeman based the correct placement of parts on comparisons between partial segments or "chainlets" of the part profiles, chosen such that it was likely that there would be only one mate with a chainlet from any other piece.

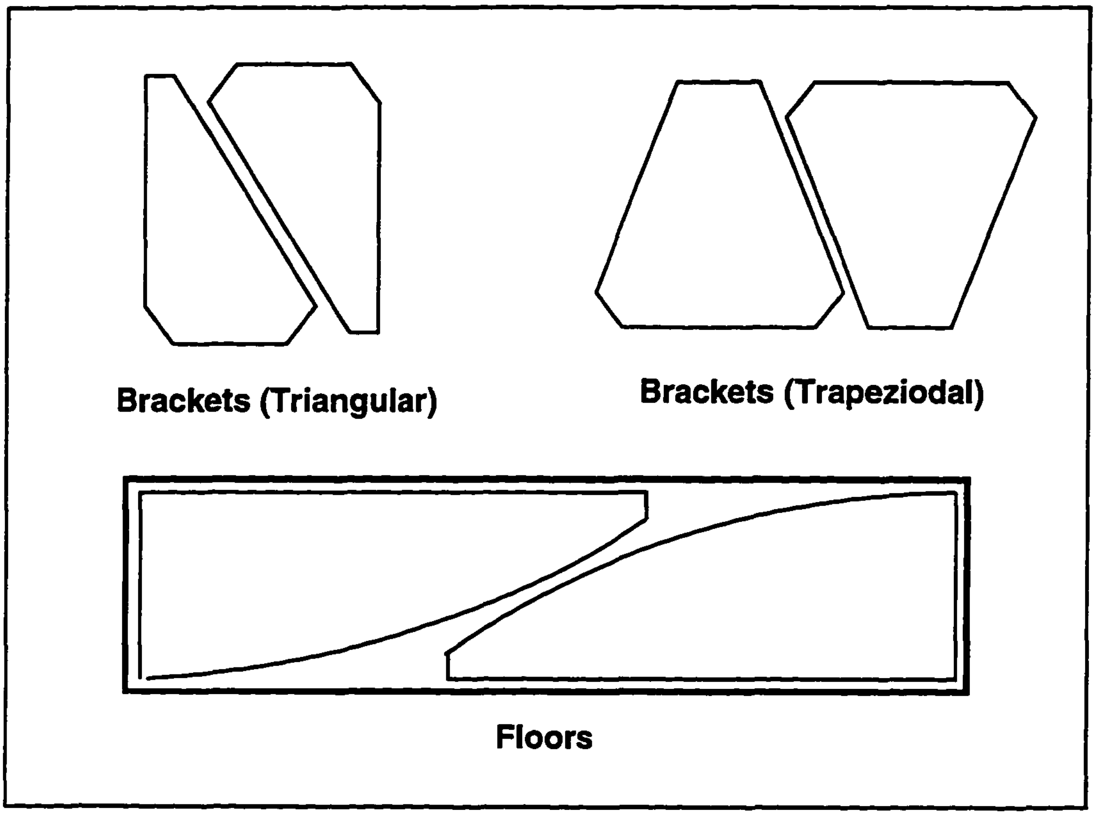


Figure 1.7 Cheok and Nee's rule based methods. Parts are classified into groups and packed into rectangular modules.

Chainlets were produced by dividing the part profiles at "critical" points, defined as inflection points and slope discontinuities in the profile [FREE78]. Since there are usually a high number of chainlets and combinations possible, a rough measure of their similarity is first established by comparing a set of orientation invariant measurable features. Examples of these "features" can be seen in Figure 1.8. This information is used to determine the most likely matches, which were then subjected to a more intensive comparison. The puzzle is assembled by adding individual pieces to a core central piece. Thus the solution grows in an outward direction.

The difficulty in applying the above approaches to nesting is that exact puzzle-like profile matches seldom exist in practice. Therefore many, but not all of the techniques discussed are of limited use.

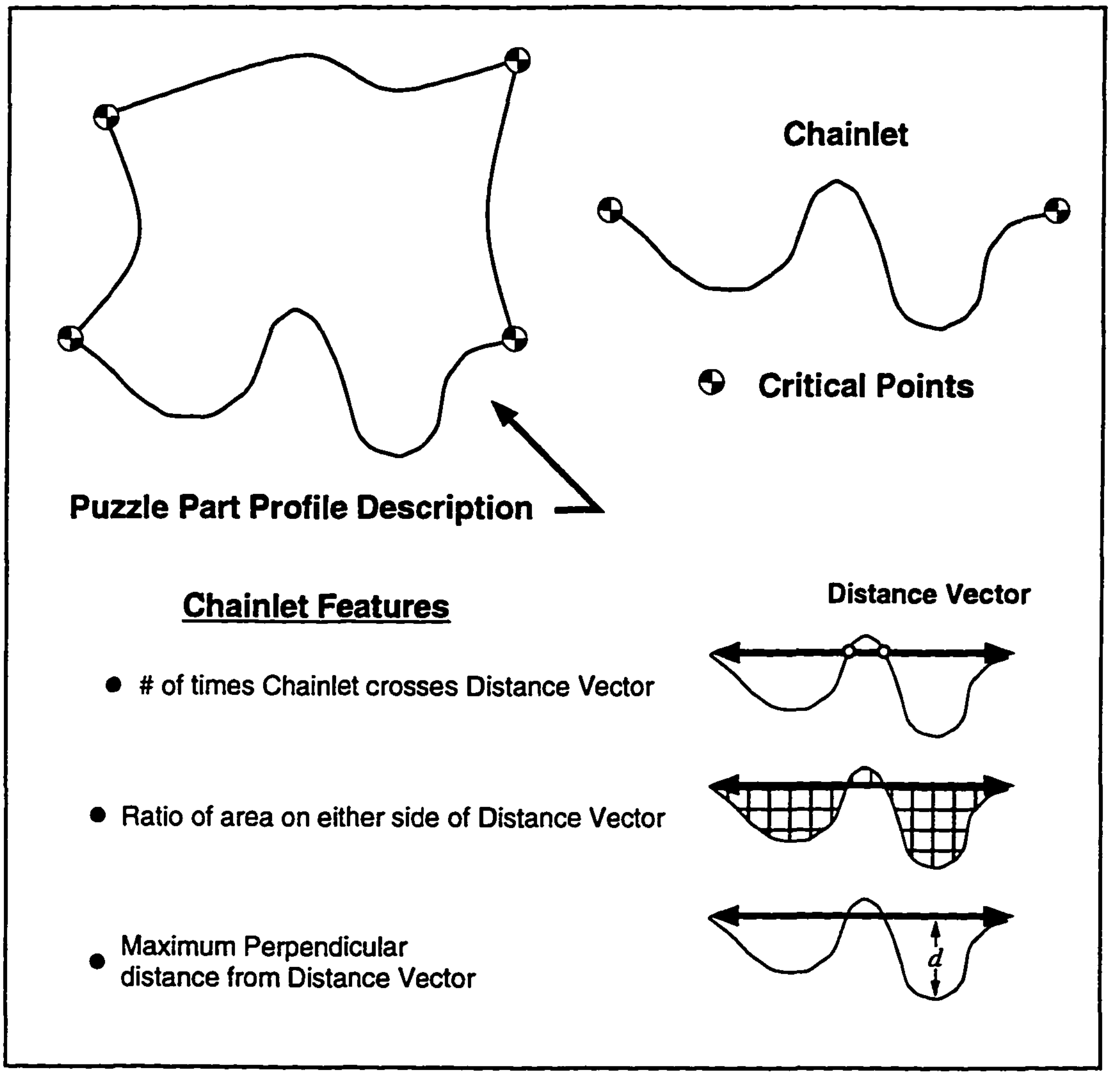


Figure 1.8 Several orientation invariant measures of shape or "chainlet features" used by Haims to solve the apictorial jigsaw puzzle problem. Chainlets are formed by dividing profiles at critical points.

However, it will be shown that the use of features for rough shape comparisons does have merit for the problem studied.

Perhaps the most comprehensive approach to solving the bin packing problem using shape is outlined by Chung and his colleagues [CHUN89]. A series of techniques is used to orient, place, and pack parts. The primary basis for placement is a shape heuristic which attempts to match concavities and convexities which exist in parts. For each part, a best fitting adjacent piece is defined for each of its four primary (90°, 180°, 270°, 360°) orientations. At each stage of the solution, the best fitting adjacent piece associated with the last placed part is tested. Unfortunately, no information is provided on how these best fit pieces are defined, and whenever the best fitting piece logic fails, a more basic angle heuristic is applied.

The angle heuristic uses an approximate polygonal representation of the irregular profiles, which must be provided by an experienced technician. For each approximating polygon, a series of outer angles is defined (Figure 1.9). Candidates for placement are determined by comparing the outer angles of a part with those of the most recently placed part. If the difference between angles falls below a threshold, that part is considered for placement. The largest of the candidates is selected as the next best fitting part. Placements are tweaked further by minor translation and rotation. A quadtree approximation of the parts is used to determine intersections and prevent overlap. Restricting parts to only four basic orientations is the key limitation of this approach which is less detrimental if nearly rectangular shapes are used. However, more complex forms generally require additional freedom if effective placement is desired.

The work presented in this manuscript represents an innovative solution technique for a general class of bin packing problems, using original methods for dealing with and reasoning about shape. The underlying objective behind this research is to go beyond simple edge and angle comparisons to develop intelligent heuristics based on more informative geometric characteristics. However, to be effective this must be done while still dealing with shape at a rudimentary level, less case

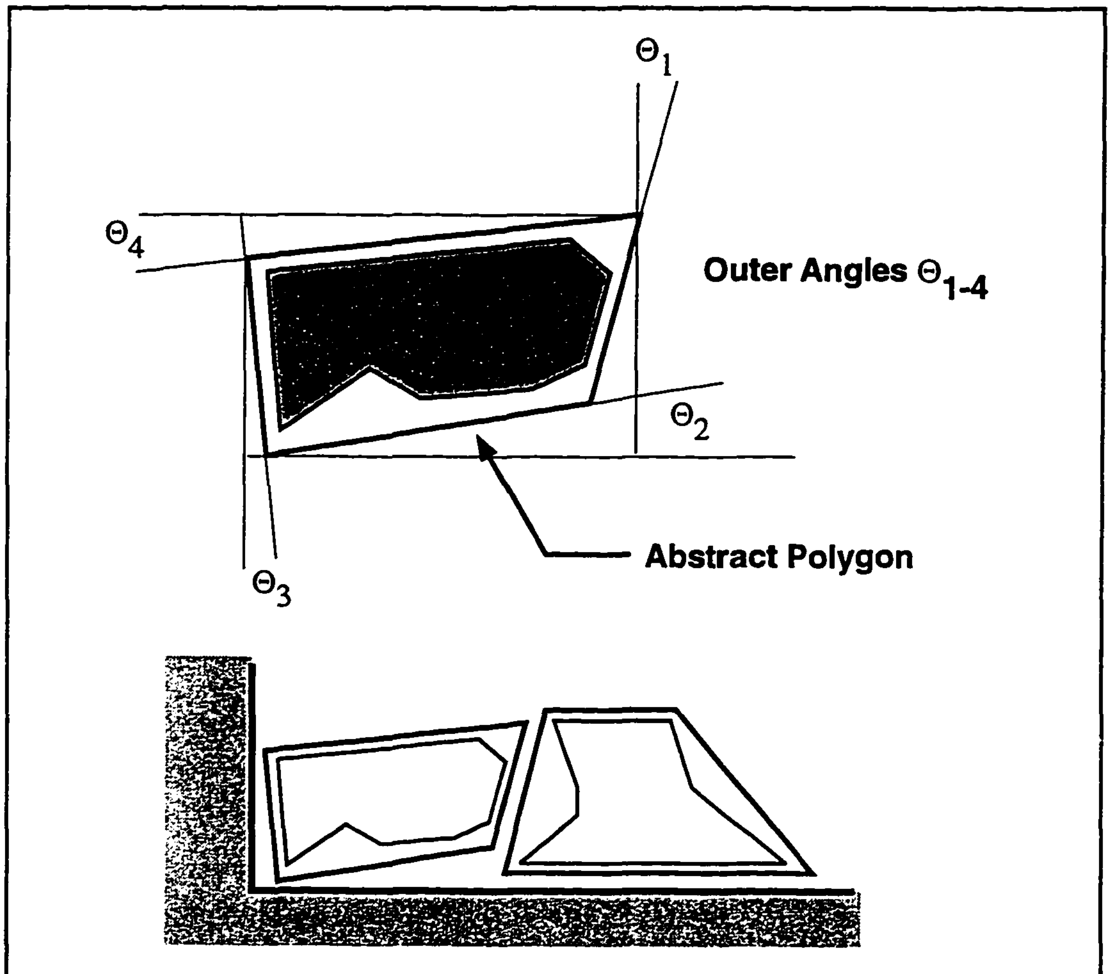


Figure 1.9 Chung's method. Profiles are placed using the outer angles of the "abstract polygon" representing each part.

specific than contemporary expert systems. This is made possible by handling the placement ambiguities present in nesting while reasoning about the solution in a way reminiscent of the puzzle problem. In the following chapters, a methodology for incorporating these concepts into a comprehensive nesting algorithm is presented.

1.5 Dissertation Layout

The remainder of the dissertation is laid out in five chapters. Chapter Two defines much of nomenclature used and summarizes the work conducted for the initial study. Chapter Three outlines the fundamental premise behind the proposed approach, and presents the basic vehicle used to describe shapes, features. Methods for extracting features from the geometries involved are also discussed. In Chapter Four, techniques for selecting and placing parts using features are demonstrated. The technique was tested on a large set of industry supplied problems to investigate its characteristics and evaluate performance. These results, the overall structure of the solution strategy, and other important control heuristics are presented in Chapter Five. The final chapter discusses conclusions drawn from the research and suggest possible avenues for future work.

Chapter 2
The Pilot Study

2.1 Introduction and Background

Interest in the 2-D allocation problem was initially generated by a request from industry to automate material requirement estimates for large offshore platform fabrication projects. A one year pilot project was conducted, and methods meeting the specific requirements of such a commercial application developed. Initially, a review of the currently available techniques was conducted, seeking solutions with the functionality to deal with highly irregular shapes efficiently, and the ability to nest on non-rectangular resources. Heuristic methods of the rule based, intelligent and expert system type (§1.4.3) were best suited to this task. Although none of the published techniques could address all concerns of the project, the method presented by Albano and Sapuppo [ALBA80] was judged best suited for needs at that time. During this initial phase of research, Albano and Sapuppo's technique was implemented and its domain of application broadened to include the more general characteristics of the industry problems involved.

The work of the pilot study is described in this chapter. In section two the method of Albano and Sapuppo is explained at length. Although a more innovative approach is presented in later chapters, several of the core elements and much of the terminology is shared with this technique. In section three, the primary adaptation permitting the use of non-rectangular stocks is detailed. Several aspects of the final solution strategy evolved from the way in which this modification was addressed.

Next, profile simplification algorithms and other techniques used to produce effective and computationally efficient representations of complex parts are presented. Finally, the overall implementation is described and its limitations discussed.

2.2 The Method of Albano and Sapuppo

As with most rule based, intelligent and rule based systems, the method of Albano and Sappuppo attempts to mimic the methods used in manual nesting. Final solutions are built by placing parts onto the resource one at a time. Two basic tasks exist for such an approach: part placement and part selection (§1.4). Part placement focuses on identifying the best location and orientation for a new part such that it generates a minimum amount of waste and does not overlap any existing parts, while part selection deals with choosing the best part to place at any given step in the solution process. The approaches used to address each task are presented separately in the following two sections.

2.2.1 Part Placement

Although determining non-overlaping positions is an important restiction during part placement, it is usually only the minimum requirement. In general the placement which will generate the least waste or trim is desired. Such a placement can often be achieved using a construct called the *No Fit Polygon* (NFP) [ALBA80]. By definition, the NFP between two parts A and B is the locus of all points where the reference point of part B may be placed such that B is touching but does not overlap A. In a more general sense, the NFP can be thought of as the path traced by the *reference point* of part B, as it translates or slides (not rotates) around part A (Figure 2.1). The reference point may be any location on the part. For simplicity, a vertex of the part profile is generally selected. This causes no loss of generality, as selecting any other point will simply translate the NFP relative to B without changing its form. Two algorithms for constructing an NFP were found in the literature.

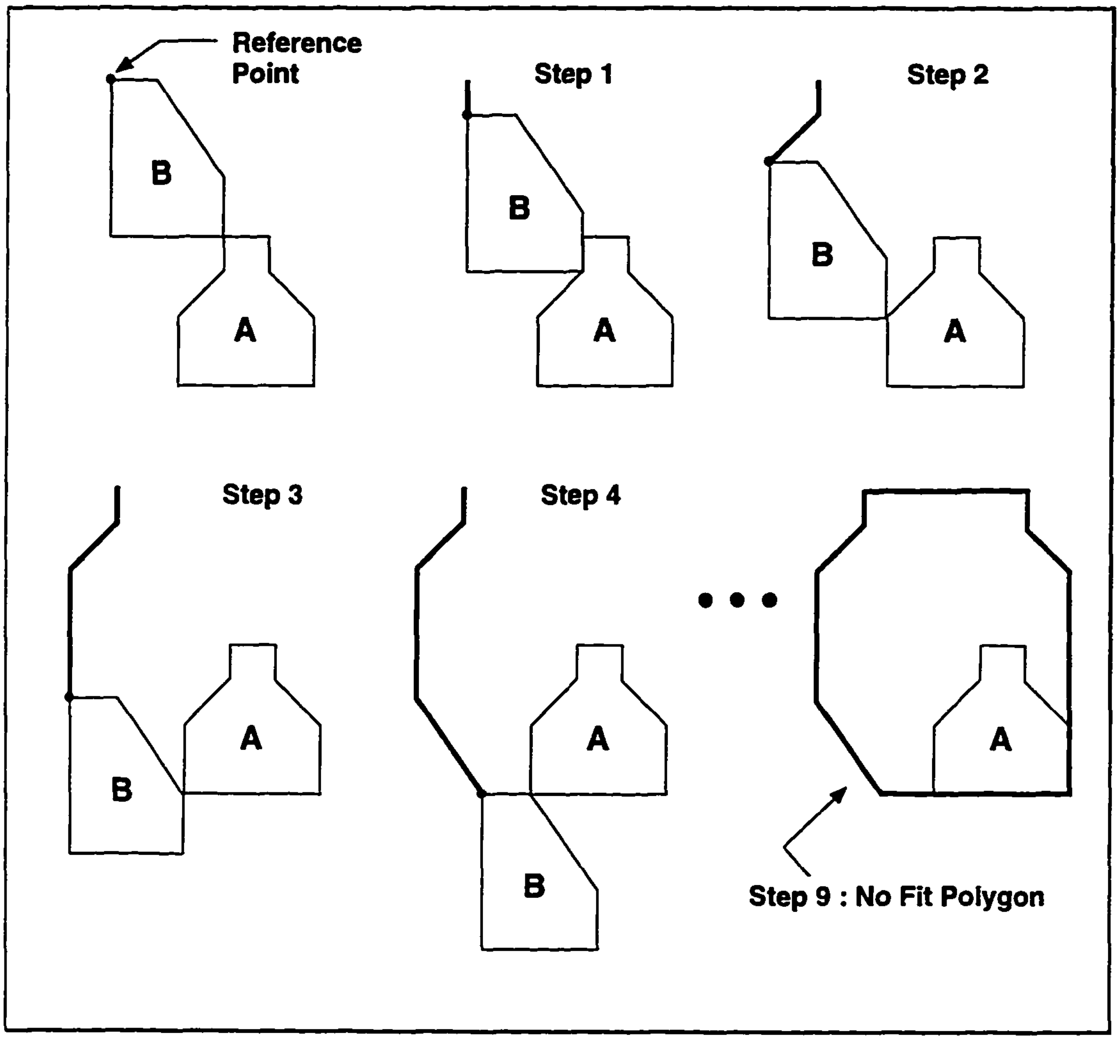


Figure 2.1 Generating a No Fit Polygon (NFP)

Lozano-Perez details a method for constructing the NFP using the known properties of convex polygons and set sum operations [LOZA83]. This formulation comes from robotics and path planning where non-intersecting positioning of objects is often of interest. If R is an object, such as a robot, moving only by translation, and A is a stationary obstacle, the set of positions for $\boldsymbol{R}$ avoiding $\boldsymbol{A}$ is the *Minkowski sum* of $\boldsymbol{A}$ and $(-\boldsymbol{R})$. The Minkowski or vector sum of two polytopes, $\boldsymbol{P}$ and $\boldsymbol{Q}$ is the set [SERR82]:

$$\boldsymbol{P} + \boldsymbol{Q} := \{\, \boldsymbol{p} + \boldsymbol{q} \mid \boldsymbol{p} \text{ is in } \boldsymbol{P} \text{ and } \boldsymbol{q} \text{ is in } \boldsymbol{Q} \,\} \qquad [2.1]$$

where + is vector addition. If the two polygons involved are convex, there is a simple algorithm for constructing the Minkowski sum. The orientation (θ) of each edge is recorded as the boundary of the two polygons are traced in a counterclockwise direction. The list of edges for both polygons is then sorted into increasing order of orientation and concatenated to produce the NFP. A similar technique using successive applications of the same algorithm may be used if the parts involved are not convex; however, a convex decomposition of the parts must be known. This substantially complicates the procedure.

A second technique for finding the NFP, described by Prasad [PRAS94], works equally well with convex and concave parts and is the method used in this research. Before outlining the algorithm, the conventions used for defining part profiles are discussed.

The profile for part A, $\boldsymbol{P}_A$, is defined by an ordered list of 2-D vertices,

$$\boldsymbol{P}_A = (\boldsymbol{V}_{1A}, \boldsymbol{V}_{2A}, \boldsymbol{V}_{3A}, \ldots, \boldsymbol{V}_{NA}), \qquad [2.2]$$

where N is the total number of vertices for that object. Vertices are coordinate pairs in the (x,y) plane, where

$$\boldsymbol{V}_i = (x_{iA}, y_{iA}). \qquad [2.3]$$

All profiles are oriented such that part material lies to the left as the boundary is transversed; thus, parts profiles are counter-clockwise. The edges of profiles are represented by the notation $\boldsymbol{E}$, where $\boldsymbol{E}_{iA}$ is the *directed* vector from $\boldsymbol{V}_{iA}$ to $\boldsymbol{V}_{(i+1)A}$. An *inward facing normal* for each edge i, $\boldsymbol{N}_{iA}$, is defined as

$$\boldsymbol{N}_{iA} = \boldsymbol{u}_z \times \boldsymbol{E}_{iA}, \qquad [2.4]$$

where $\boldsymbol{u}_z$ is the unit vector in the positive z direction. It is also useful to classify each vertex as either convex or concave. The following definition is used.

$$\begin{aligned} &if\,(\ (\boldsymbol{E}_{(i-1)A} \times \boldsymbol{E}_{iA}) \cdot \boldsymbol{u}_z \geq 0\ )\ \mid\ \boldsymbol{V}_{iA}\ is\ convex \\ &\quad else \\ &\boldsymbol{V}_{iA}\ \ is\ concave \end{aligned} \qquad [2.5]$$

When the meaning is clear from the context, the subscript "A" identifying the part may be dropped from all of the above notation.

The NFP of part B with respect to part A is produced as B's profile slides along the stationary boundary of A in a counterclockwise fashion. The path taken by part B's reference point can be decomposed into a series of steps which may be determined using the technique described below.

An initial position for B, such that it intersects but does not overlap A, is provided by co-locating the leftmost lowest vertex of A with the rightmost highest vertex of B. The intersection points are referred to as the *contact vertices* and are denoted as

contact vertex A : $\boldsymbol{CV}_A = \boldsymbol{V}_{iA}$

contact vertex B : $\boldsymbol{CV}_B = \boldsymbol{V}_{jB}$

where i and j reference the appropriate vertex within the ordered list describing each part profile (Figure 2.2). In this position two translation directions are possible for part B, such that it slides along but does not overlap part A. The direction of movement, or *movement vector* $\boldsymbol{S}$, is given by

$$S = E_{iA} \quad | \quad iff \quad (E_{jB} \times E_{iA}) \cdot u_z \geq 0 \qquad [2.6]$$

otherwise

$$S = -E_{jB}$$

where the maximum distance of translation is indicated by the corresponding edge length of S.

To determine the extent of movement, a copy of the ray S is extended from all vertices of B and the shortest distance to intersection with an edge of A recorded (Figure 2.3). A similar procedure is applied for the vertices of Part A. However, copies of $-S$ are used, and intersections with the edges of B are calculated.

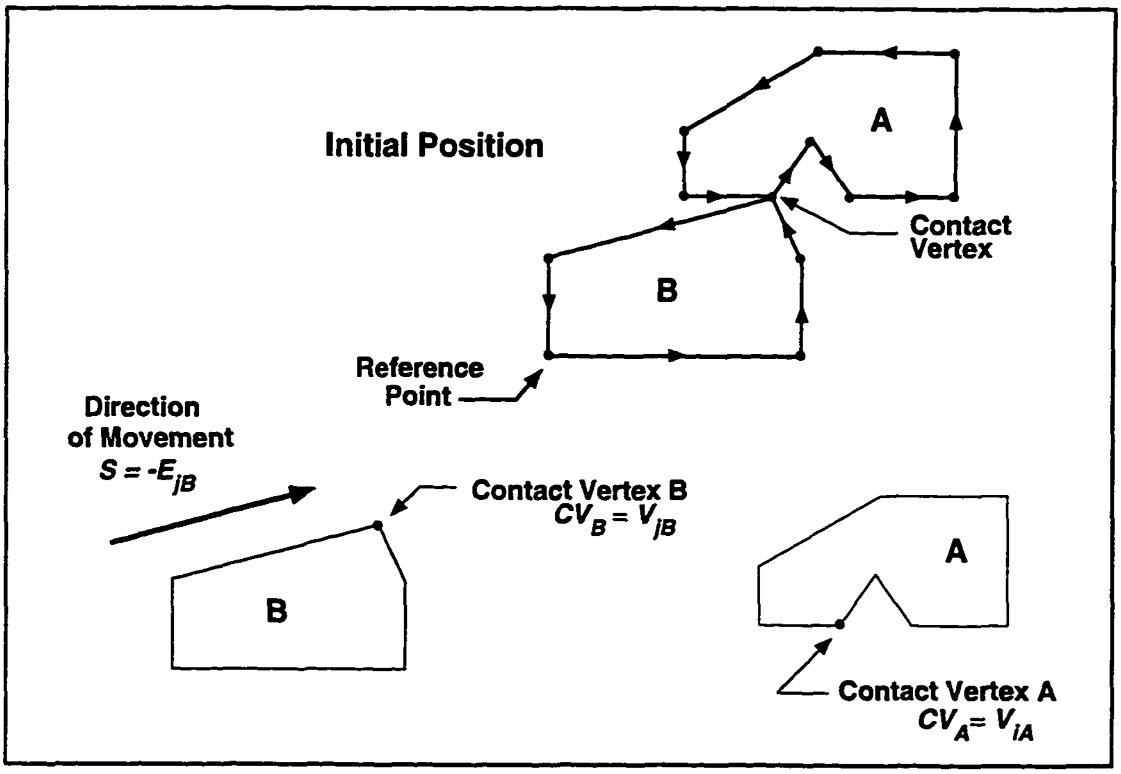


Figure 2.2 Initial positioning to generate the NFP between Parts A and B showing the direction of movement and contact vertices.

If no intersections occur within the prescribed limit, B is translated by the maximum length possible, and depending on the movement vector used, one new contact vertex is identified from

$$CV_A = V_{(i+1)A} \qquad [2.7]$$

or

$$CV_B = V_{(j+1)B} \cdot \qquad [2.8]$$

The second contact vertex remains unchanged.

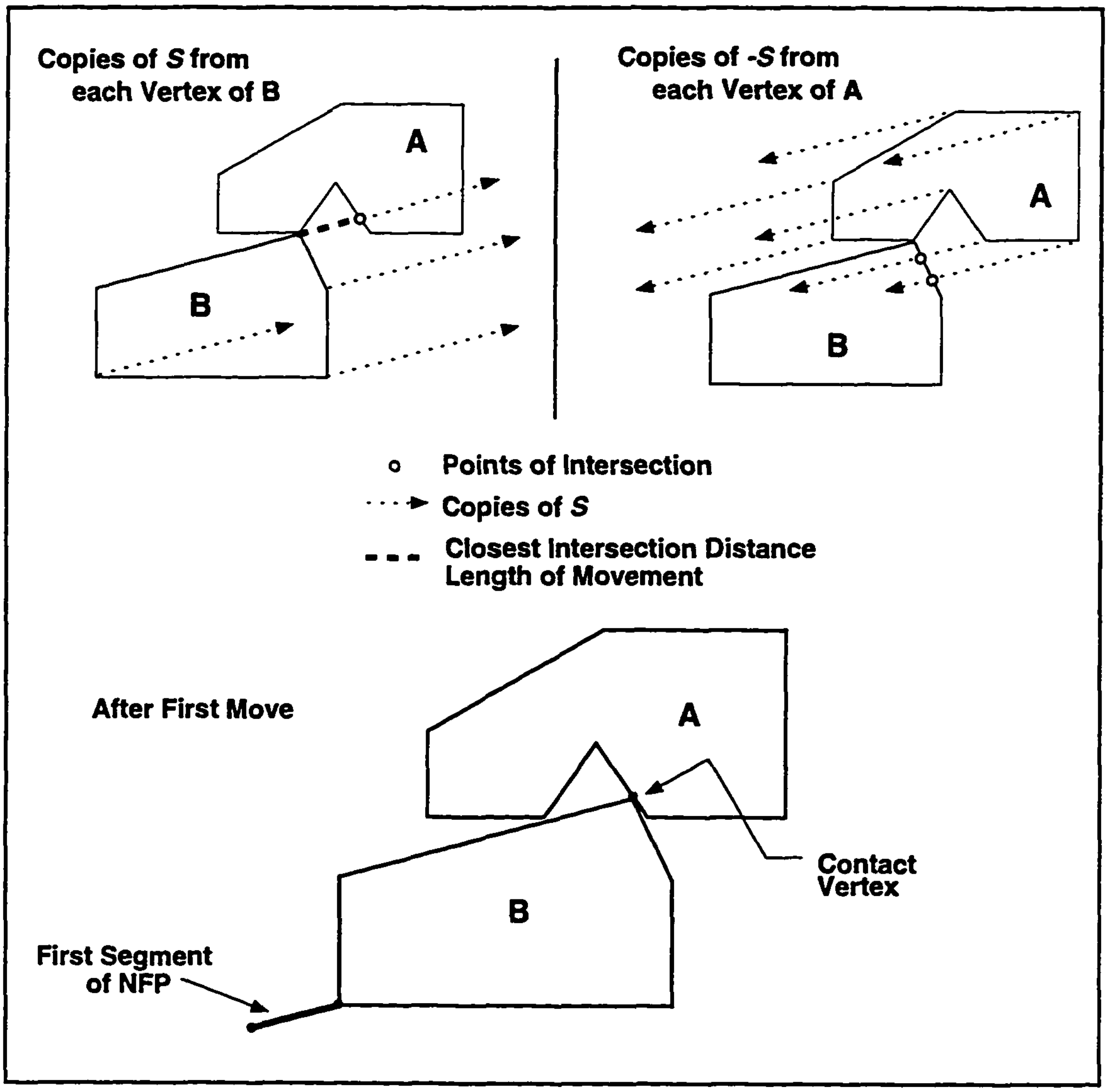


Figure 2.3 Determining the extent of movement at each stage as part B moves around part A in a counterclockwise direction.

When a valid intersection was detected along the movement vector part B is translated by that distance dictated by the intersection. The vertex from which the shortest movement vector S (or $-S$) was recorded becomes the new contact vertex for that object. The location of the intersection on the other part's edge is also saved, and a new vertex inserted at that location. This point becomes the new contact vertex for that part. A new direction of movement is formulated with these contact vertices, and the process repeated until the reference point of B eventually returns to its initial position.

Since more than two parts are normally involved in a layout, part A may represent a single part profile as above, but is most often the cumulative profile of all of the parts already placed on the stock sheet, or *merged profile* (Figure 2.4). The merged profiles of layouts consisting of numerous parts may become complex; consequently, a large computational expense is incurred in determining the intersections between each direction vector S and all edges involved. Several measures are

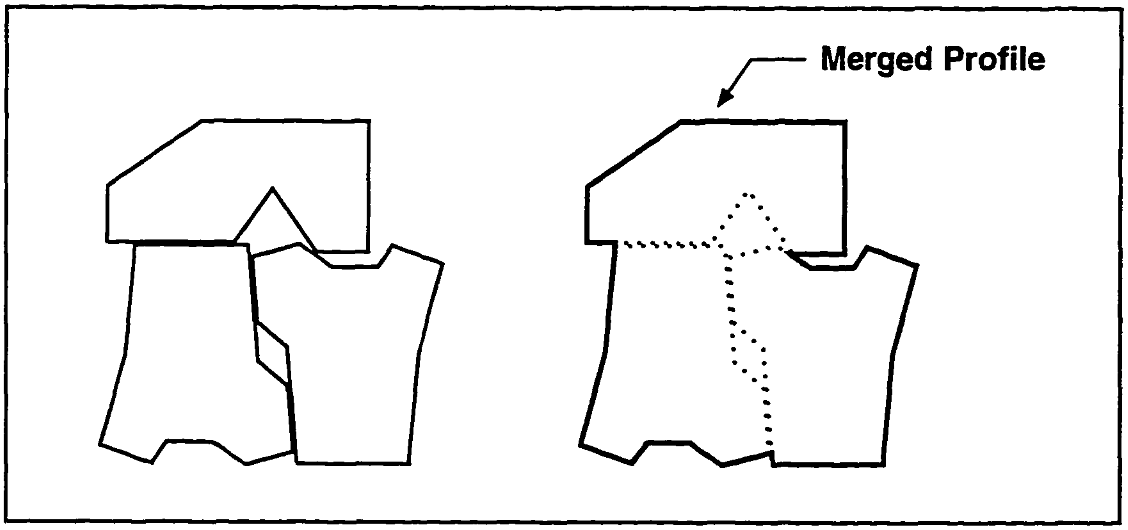


Figure 2.4 A group of parts and their associated Merged Profile.

implemented to eliminate calculations, which are unnecessary based on the non-manifold nature of the involved profiles.

All edges not facing the the movement vector may be eliminated, as a nearer intersection with some other edge of the part facing the direction of motion is always guaranteed with closed profiles. Based on reasoning analogous to that used in back face rejection hidden line removal algorithms [FOLE84], the edges of part A may be eliminated from intersection calculations using the metric

$$if\ (\mathbf{S} \cdot \mathbf{N}_{iA} \leq 0)\ |\ eliminate\ \mathbf{E}_{iA}. \qquad [2.9]$$

Movement vectors may be eliminated if they point into the part, since, as with the edges eliminated above, a closer intersection is always provided by the remaining entities. Elimination of movement vectors is determined by examining the vertex from which they originate (Figure 2.5). Two cases are possible. For the concave vertices of part B the criterion is

$$if\ (\mathbf{S} \cdot \mathbf{N}_{(i-1)A} > 0)\ OR\ (\mathbf{S} \cdot \mathbf{N}_{(i)A} > 0)\ |\ eliminate\ \mathbf{V}_{iA} \qquad [2.10]$$

while for convex vertices it is

$$if\ (\mathbf{S} \cdot \mathbf{N}_{(i-1)A} > 0)\ AND\ (\mathbf{S} \cdot \mathbf{N}_{(i)A} > 0)\ |\ eliminate\ \mathbf{V}_{iA} \qquad [2.11]$$

Similar criteria for the edges of part B and the vertices of part A may be formulated by substituting $-\mathbf{S}$ for $\mathbf{S}$ in the above equations. The cost of calculating the NFP can be substantially reduced in this way.

To position a new part B onto the nest such that it is touching but not overlapping any of the existing parts, the reference point of B will be placed somewhere on the NFP. A second condition, which must be satisfied by B's placement, is that it must fall totally within the boundaries of the resource or stock material. In order to meet this constraint, the *allocation region* of part B on the resource is determined. The allocation region for a part in a given orientation is simply the area in which the reference point on B may be placed, such that B is contained entirely within the resource. This area, shown in Figure 2.6, is easily determined for a rectangular resource, given the minimum and maximum x and y extents of the part relative to its reference point. By selecting a point on the NFP which is contained in the allocation region, it can be insured that both conditions are satisfied.

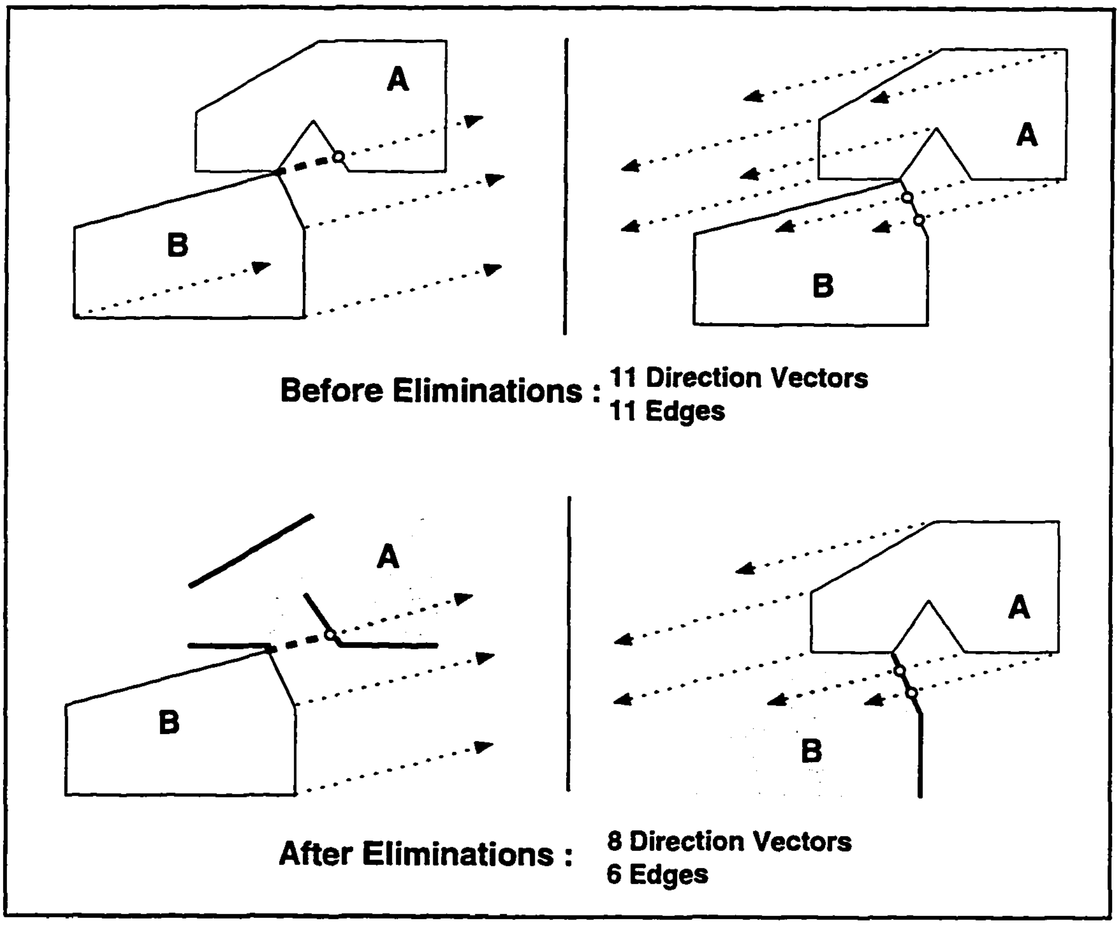


Figure 2.5 Savings through elimination of edges and vertices during NFP intersection calculations

In practice, an NFP and allocation region must be calculated for each candidate orientation of a part. For a given orientation, all acceptable locations of the part are determined by finding those portions of the NFP falling within the allocation region. This is done by clipping the NFP on the rectangular boundary of the allocation region using a 2-D Cohen-Sutherland algorithm [FOLE84]. A *placement policy* is then used to select the location which is most appropriate for this orientation. In Albano and Sapuppo [ALBA80], a leftmost lowest placement policy was applied, causing parts to be packed in the lower left-hand corner of the stock plate. Other placement policies are equally valid. With a position selected for each orientation, the placement policy is applied once again to

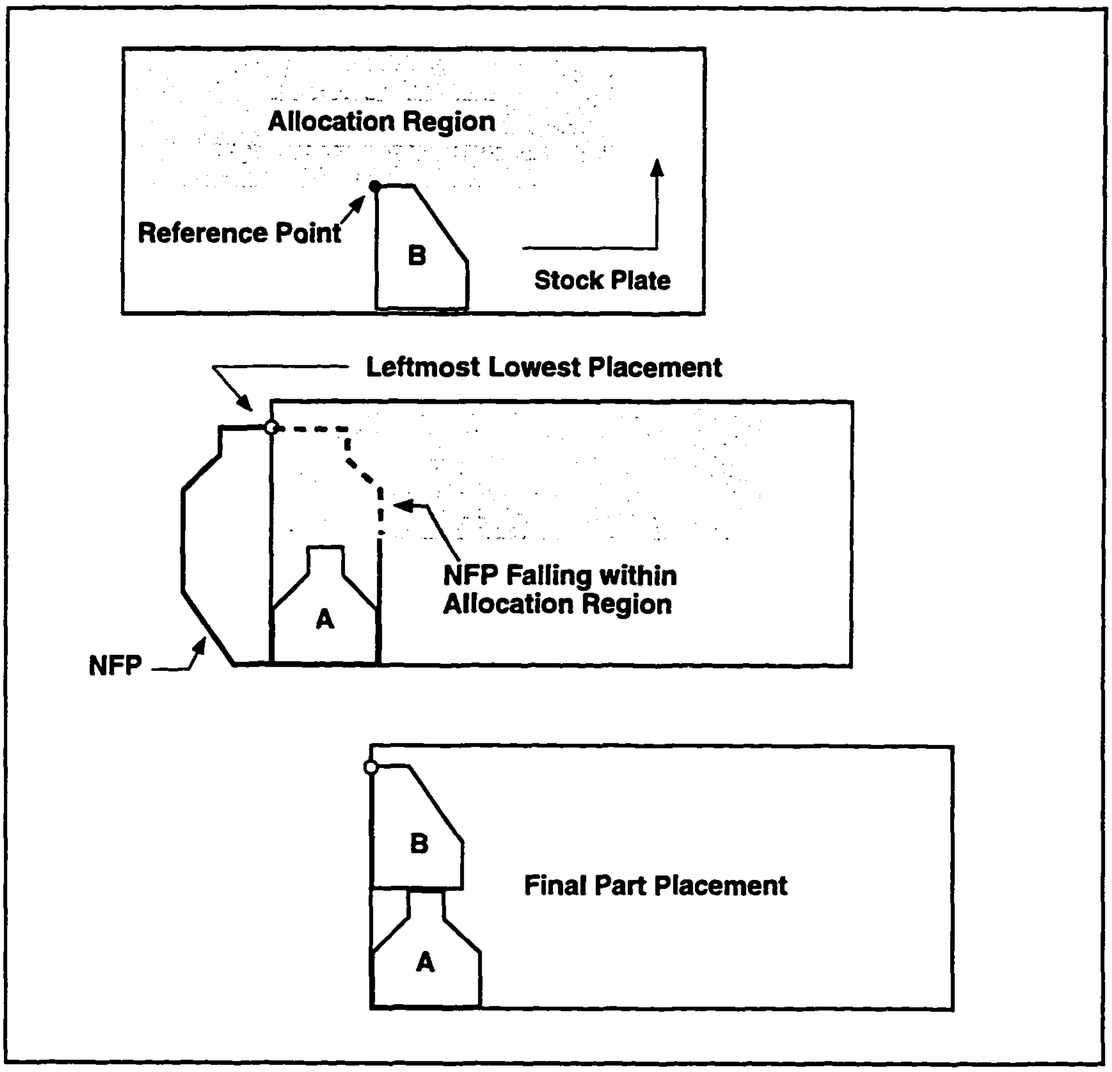


Figure 2.6 Part placement withing the allocation region

select that candiate with leftmost lowest extention. The quality of the solution generated depends greatly on the number of distinct orientations tested for each part.

2.2.2 Part Selection

The other basic task of the nesting heuristic involves selecting the most appropriate part for placement at any given stage in the solution process. The tree structure of Figure 2.7 is a graphical representation of the various possibilities. Nodes on the tree are referred to as *allocations*, and each represents an *intermediate solution* to the layout problem. The *level* of an allocation is denoted by the number of placed parts it contains. At each stage of the nesting algorithm, a new intermediate solution or *successor* can be generated from any node containing unplaced parts.

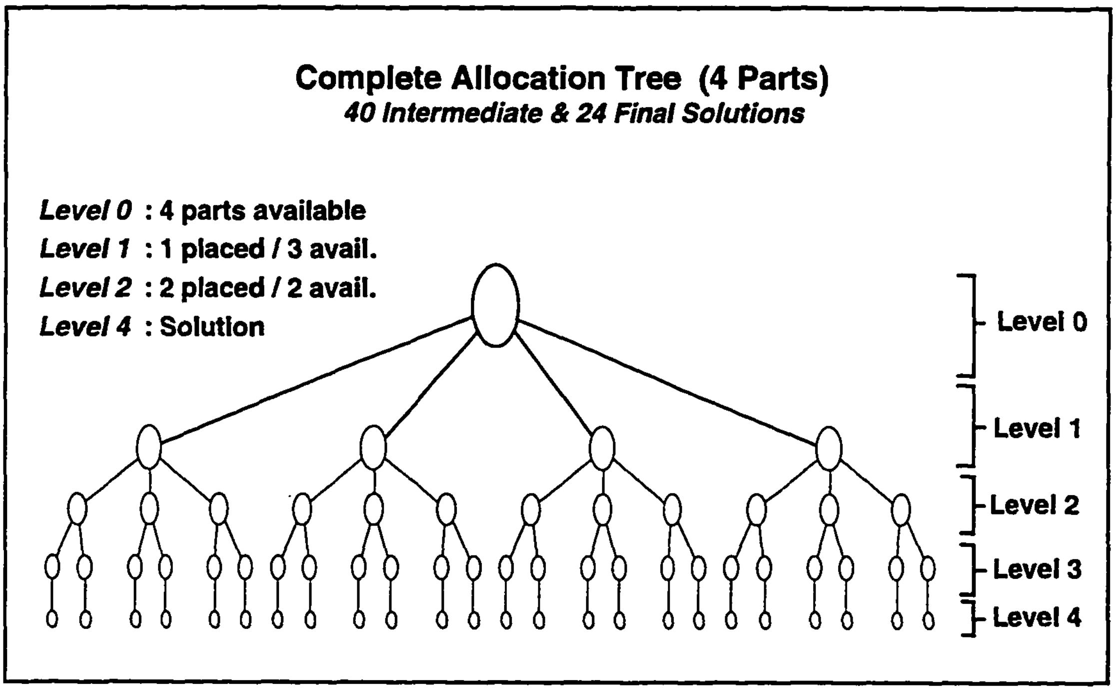


Figure 2.7 A complete allocation tree for 4 parts with only one orientation allowed.

A single part can produce an infinite number of successors to a node, as each of its possible positions (x,y,θ) will correspond to a new intermediate solution. To reduce the total number of nodes, each part is allowed to generate only one successor, corresponding to the placement generated by the NFP / leftmost-lowest method of the previous section. Even when restricted in this way, a large number of intermediate solutions can be generated from even a small group of parts (Figure 2.7).

The majority of intermediate solutions are not of interest, as only those nodes lying on a path between the initial and optimal final node are necessary to generate a solution. The total number of intermediate solutions produced can be substantially reduced if only these required nodes are generated (Figure 2.8). Albano and Sapuppo propose a *search technique* for transversing the allocation tree to produce such an optimum search path [ALBA80].

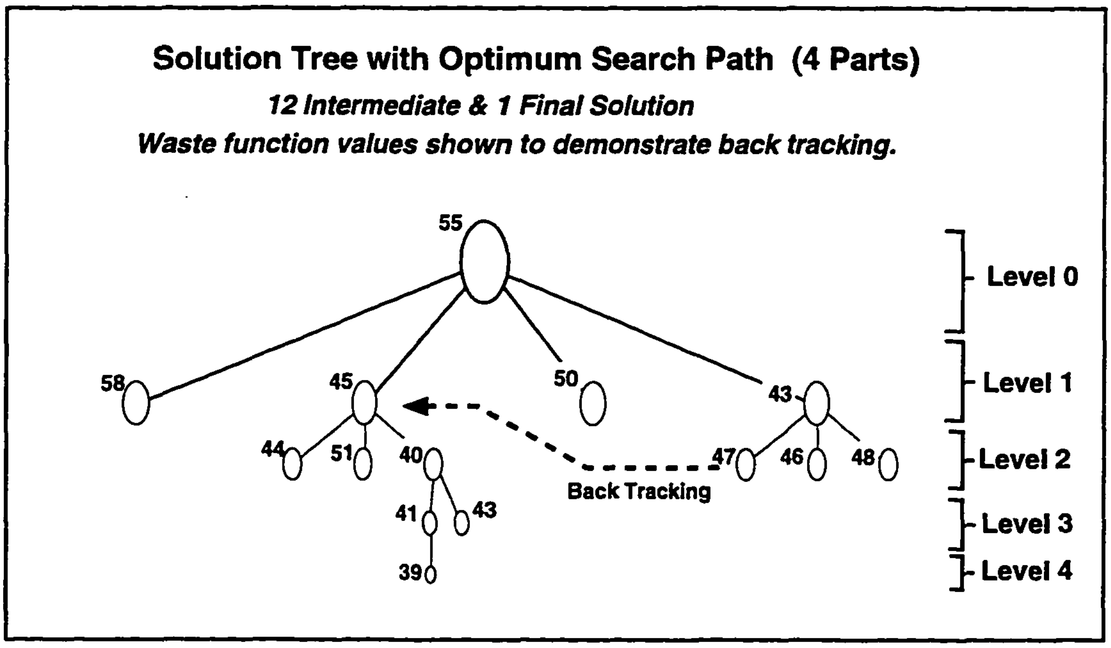


Figure 2.8 A trimmed allocation tree showing the optimum search path

Using this method, at each level only successors to the "best" node are produced. Selection is based upon minimizing a potential *waste function* which consists of two components: *true waste* and *future waste.* True waste is the area of the gaps trapped between parts which is not available for further nesting (Figure 2.9). Future waste is an estimate of the waste to be incurred through nesting of the remaining parts. In Albano and Sapuppo, future waste is defined as a fixed percentage of the total area of remaining parts. This effectively biases the solution towards the early placement of larger parts. Backtracking to a previous level is also permitted, if the current solution path produces less desirable results than a previous node. The degree of backtracking allowed is controlled by the *expansion band,* a parameter which limits the total number of levels the solution path may jump backward.

Due to the large number of parts involved in many problems, other restrictions are implemented to limit the total number of intermediate allocations generated and stored during the solution process. An

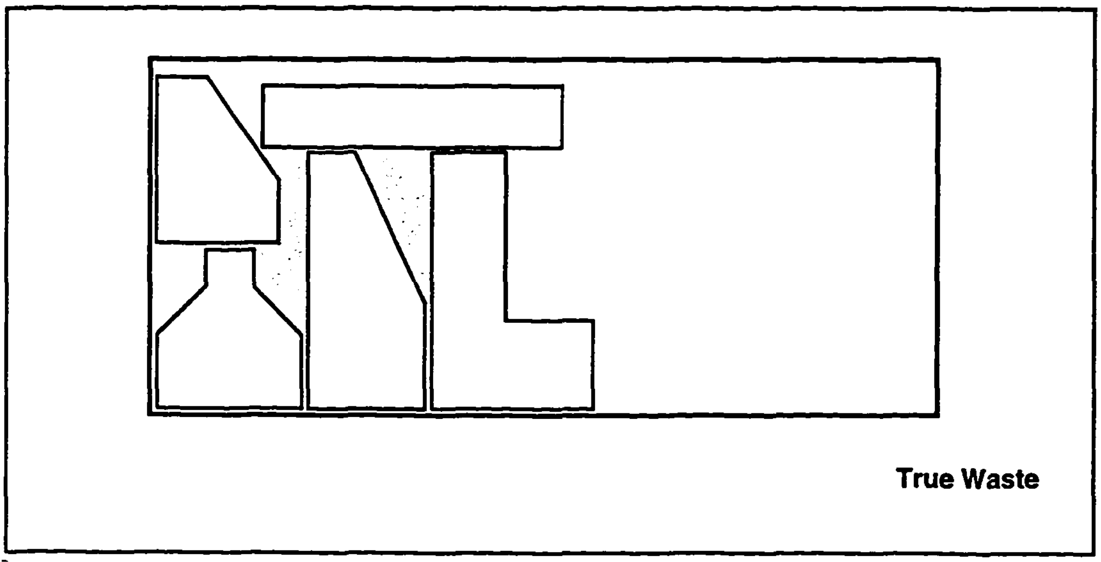


Figure 2.9 The true waste associated with a layout

upper bound on the number of successors produced at each stage is set using the parameter *N_SUCCESSOR*. The leftmost extension of each successor's most recently placed part is used to determine which layouts are retained. The number of intermediate allocations stored from previous stages of the solution is also limited. These nodes are only of potential use during backtracking; those extending beyond the expansion band are automatically eliminated. Furthermore, only a set number (*MAX_GENERATED*) are retained based on lowest potential waste. A more detailed explanation of the theory and implementation of the search technique can be found in [ALBA80].

2.3 Adaptations

In order to deal more effectively with difficulties encountered in many industrial nesting applications, several substantial changes were made to Albano and Sapuppo's method. These extensions and enhancements produce better part placements, handle complex parts in a computationally efficient manner, and allow the technique to nest within voids. Each of these improvements is detailed in the following sections.

2.3.1 Irregular Resources

The method described in the previous section is designed to lay out parts onto an infinitely long rectangular stock. Added functionality is required if parts are to be nested onto finite resources with generic irregular boundaries. Such situations occur in the production of leather goods where animal hides are seldom rectangular, and may also occur in the offshore platform and ship building industry, where small pieces are often nested within the voids of larger parts to increase material usage.

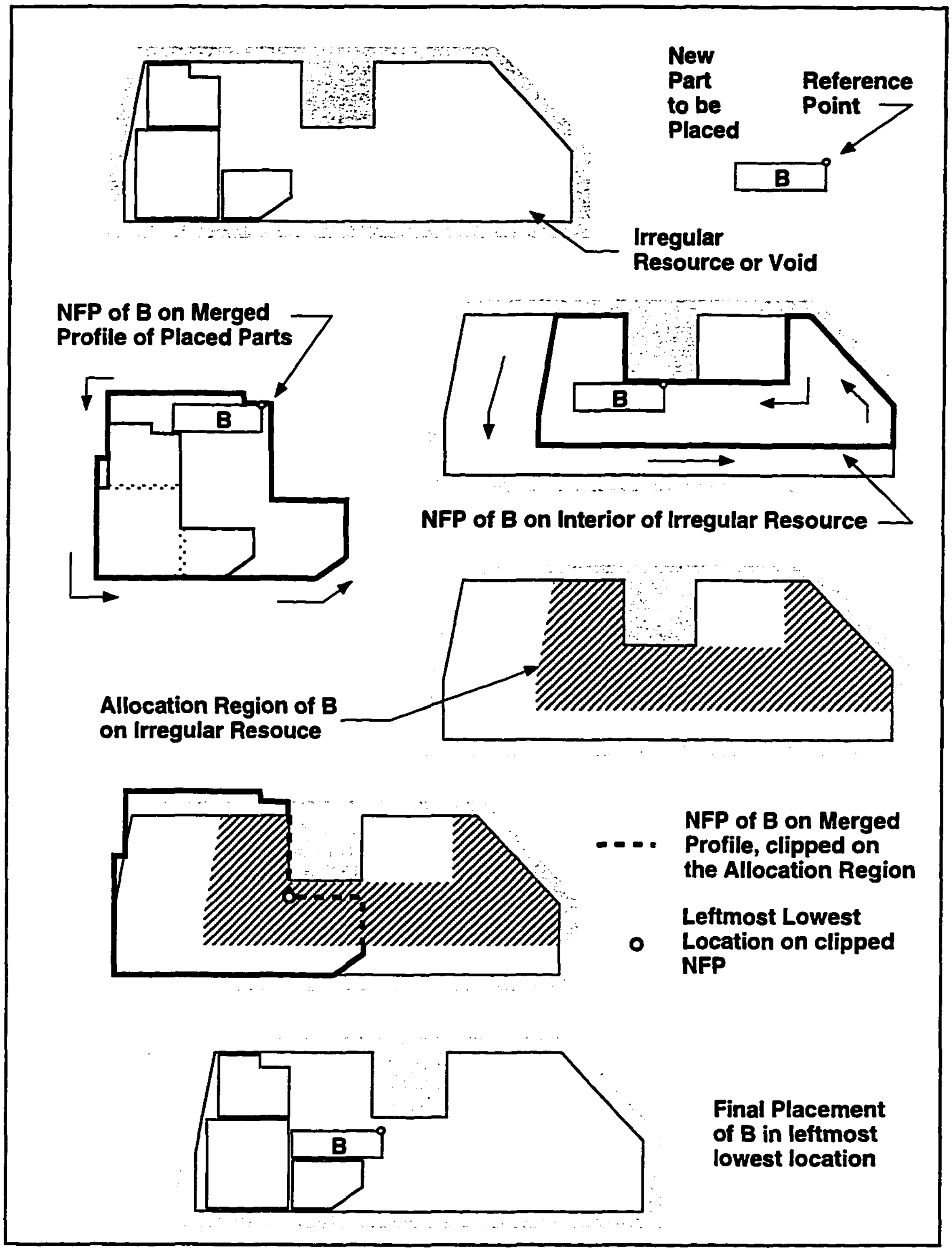


Figure 2.10 The method of Albano and Sapuppo applied directly to irregular resources. Note that the allocation region is the area enclosed by the NFP of B on the interior boundary of the resource.

If Albano and Sapuppo's method is to be directly applied to nesting on irregular resources, two NFP's must be calculated for each part placement. As before, an NFP of the part on the merged profile of all parts previously placed is constructed. In order to determine the part's allocation region, an additional NFP must be constructed on the interior boundary of the resource (Figure 2.10), since the simple bounding box method used previously (§2.2.1) is no longer valid. Furthermore, clipping of the merged profile NFP is also more complicated as the new allocation region is no longer rectangular and may be non-convex.

The proposed heuristics for nesting on irregular resources are similar to those used for a rectangular resource; however, the allocation region and NFP are combined into a single construct called the *Internal No Fit Polygon* (INFP). Analogous to the NFP, the INFP is the path traced by the reference vertex of a part as the part slides in contact with the interior of a region such that the part is totally contained within and is just touching the boundary. To effectively eliminate the construction and subsequent clipping on the allocation region, the INFP must be performed on the *remaining resource*. The remaining resource refers to the area within the void profile unused by the previously allocated parts (Figure 2.11). The leftmost lowest placement now conveniently corresponds to the leftmost lowest vertex of the INFP. With this new procedure for placing parts, the strategy proceeds as before.

To construct the INFP using the same NFP algorithms discussed earlier, two provisions must be made. First, the irregular resource must be formatted as a part profile. The is easily accomplished by defining the boundary such that material lies to the left as the profile is transversed, as is needed with parts. If all irregular resources are viewed as voids, this is achieved by a clockwise boundary orientation. Edges and normals may then be defined as previously noted. Second, an initial placement such that the part is totally contained within the void without overlapping the boundary must exist. The original contact vertices required to initiate the NFP algorithm, may be generated from such a

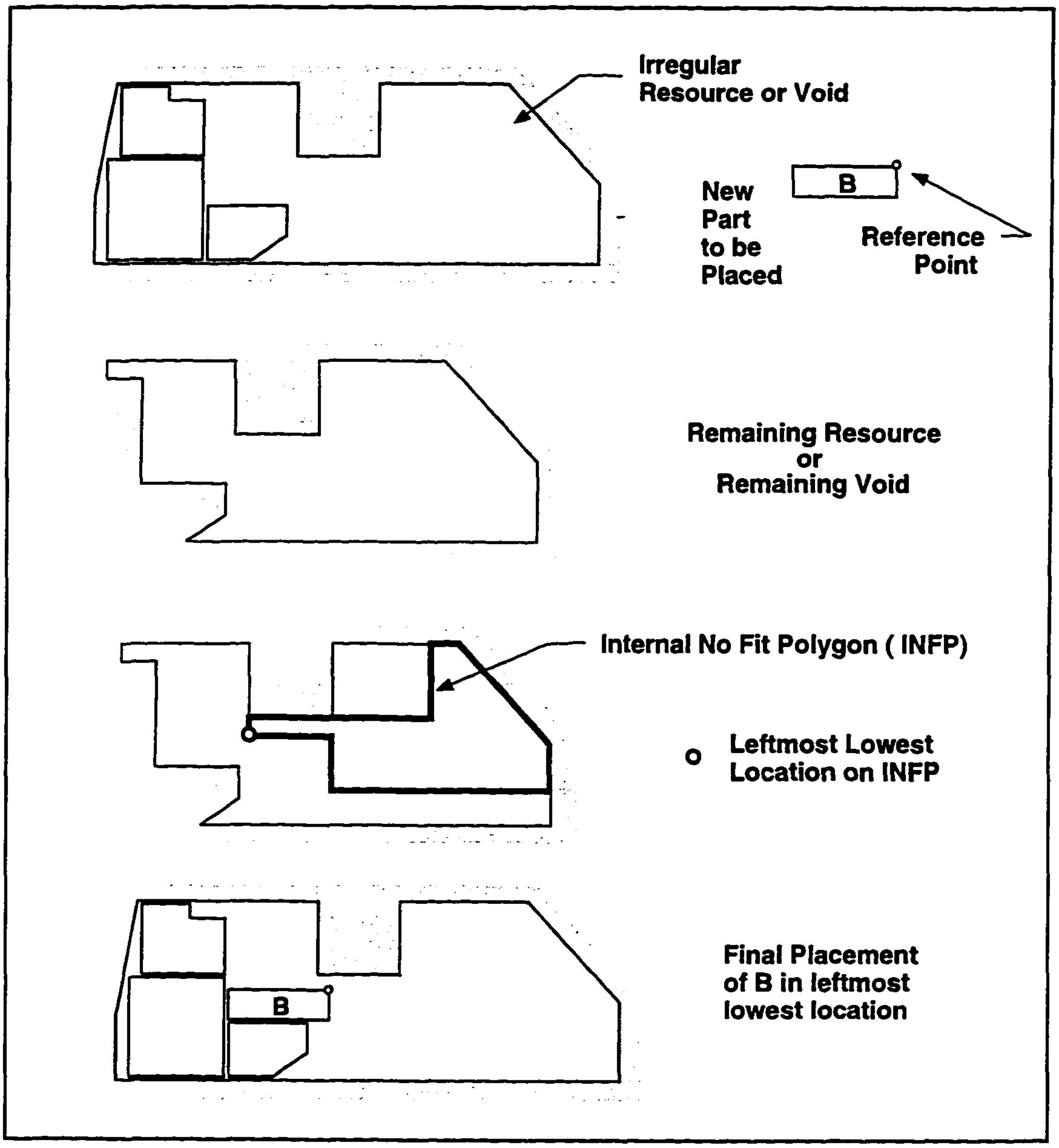


Figure 2.11 Part placement in voids using the Internal No Fit Polygon

position, by shifting the part until an intersection is found. If the void is convex, the initial placement is simple to calculate. For non-convex shapes, however, the initial placement selection is not as simple and a decomposition of the void into convex regions is necessary. Candidate locations are generated by a recursive subdivision algorithm as described in Pavlidis and Feng [PAVL78]. The basic steps are:

(1) Determine all concave vertices of the void profile and the distances between them.

(2) If non-adjacent, concave vertices exist, divide the region along the line connecting the two closest non-adjacent concave vertices. If only adjacent, concave vertices exist, divide the region along the line bisecting the interior angle of one concave vertex.

(3) Return to step (1) and apply the procedure recursively to the two regions produced in step (2).

This process continues until all remaining areas are convex polygons (Figure 2.12).

Initial placements for a part are determined by collocating the center of the min/max box of the part with the centroid of a convex subdivision. Due to the nature of the INFP construction method, necking and small notches within the remaining void may cause some areas of the resource to be inaccessible as the part slides along the boundary of the resource. Consequently, each starting location may yield a distinctly different INFP and part placement (Figure 2.12). Therefore, the candidate regions corresponding to the desired placement policy (leftmost lowest) are tested first. To increase the likelihood of successful nonoverlapping initial placements, subregions with small areas and dissimilar min/max box aspect ratios, relative to the part in question, are eliminated from consideration. The initial location suggested may also be adjusted slightly when only minor overlaps are detected between its profile and the boundary of

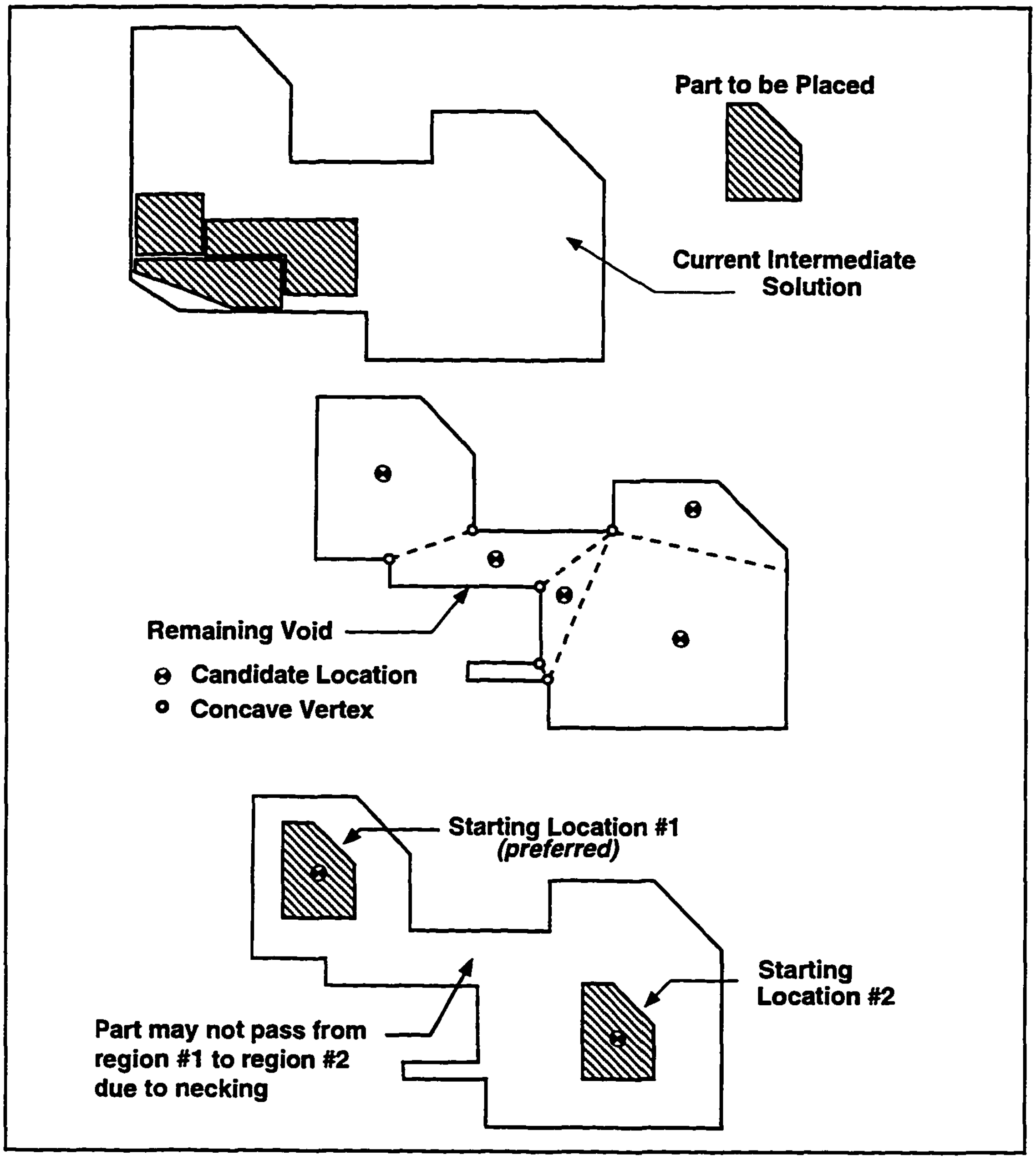


Figure 2.12 Generating Initial Placements through convex decompositon

the void. Once totally contained within the void, construction of a valid INFP is assured.

2.3.2 Part Simplification

Calculation of NFP's and INFP's can account for 80 to 90 percent of the computational expense associated with this method of solution. The processing time required for each of these constructs is a direct function of the complexity of the profiles involved.

For the pilot study, profiles were restricted to circular arcs and straight line segments. For convenience, all arcs were approximated to a desired accuracy by inscribed or circumscribed chords as appropriate. Within industry, profiles with greater than 60 to 70 line segments were commonly encountered with even the coarsest circular approximations. Parts of this nature significantly increase the computational expense of the NFP calculations.

Most complex shapes can be reduced to simpler, approximate forms without adversely affecting the quality of the nest produced. A part simplification algorithm using a *modified convex hull approach* was developed to achieve this goal.

Each part is processed by first constructing its *convex hull* [BOWY93], which reduces the complexity of the object but is often an inefficient approximation of the shape. This is easily demonstrated by the example of an "L" shaped bracket, where substantial waste would be incurred if the convex hull approximation were used (Figure 2.13).

Further simplification is achieved through analysis and alteration of the *concave region* profiles. The boundaries of these regions, which are exterior to the part but interior to the convex hull, are described by a section of the part profile and a single *closing hull edge*. The closing hull edge is that member of the concave region profile which is also a component of the convex hull boundary.

The concave regions may be incorporated into the simplified part profile if either of two conditions are met. The first criterion is based on the size of the concave region. If this area is small relative to the size

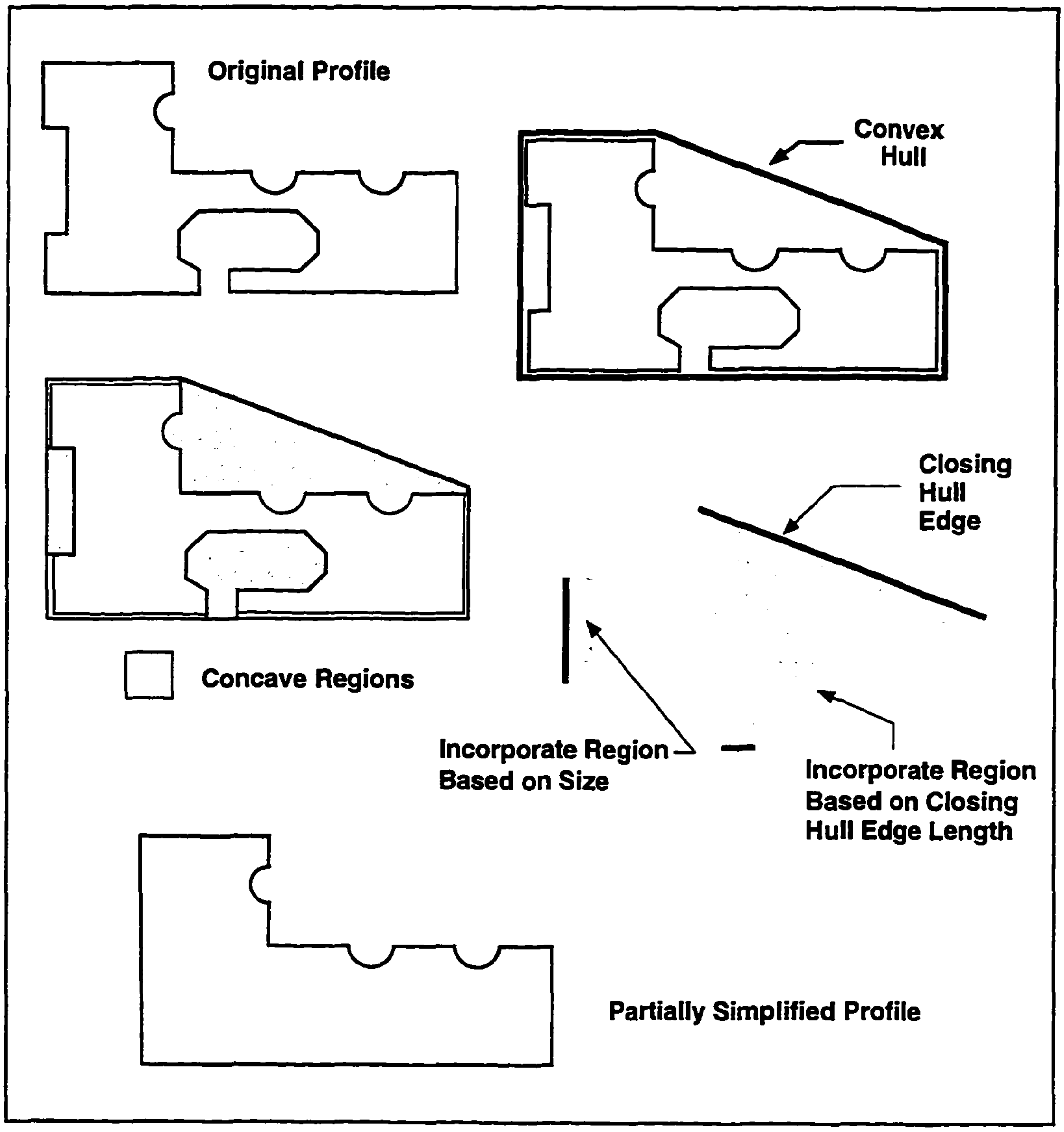


Figure 2.13 The modified convex hull part simplification procedure showing the total incorporation of concave regions

of the parts to be nested, the waste incurred by including it in the simplified profile will be negligible. The second criterion is based upon the length of the closing hull edge. If this length is small, it is unlikely that a part could enter this region through the NFP placement mechanism. When either condition is satisfied, the profile is simplified by eliminating the original part edges associated with the concave region and replacing them with the single closing hull edge.

Concave regions which cannot be incorporated into the approximating profile using the two above criteria may still be simplified using an approach similar to that used in finding initial placement locations within voids. The area is recursively subdivided by lines connecting closest concave vertices (Figure 2.14), as discussed earlier. These *dividing lines* are considered as closing hull edges and their associated areas as "pseudo" concave regions. The appropriate dividing lines may be incorporated into the simplified profile based upon the two elimination criteria previously given. A demonstration of the entire method is shown in Figures 2.13 and 2.14. For this particular profile, the algorithm produces a reasonable facsimile while reducing the complexity of the part by eighty percent (down from 33 to 6 edges).

2.3.3 Optimal Part Orientation

Another concern involving the NFP is the selection of part orientations. For an optimal solution, all orientations of a part should be tested, a proposition which is not feasible. The computational expense of calculating an NFP for each orientation effectively eliminates this as an option. For this reason, each part is limited to only four possible orientations. Results from the preliminary study indicate that aligning the

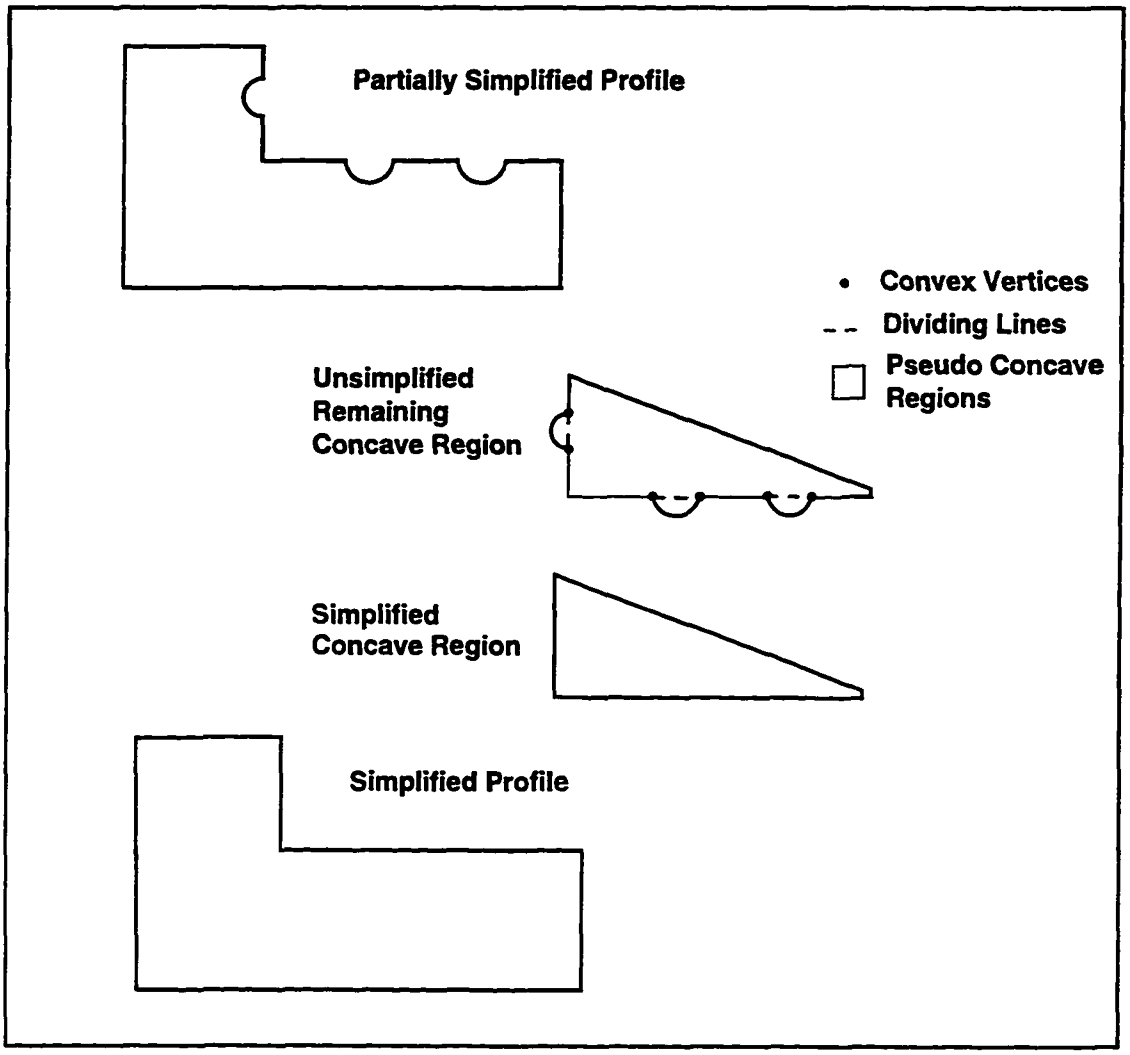


Figure 2.14 The modified convex hull part simplification procedure showing simplification of a concave region

part's MER with the boundary of the rectangular stock is a good base orientation. The three other orientations are provided by successive ninety degree rotations. For voids, a base orientation is provided by aligning the part's MER with the MER of the void boundary (Figure 2.15). The suitability of these orientations depends greatly upon the accuracy of the MER.

As a further aid in placement, the four orientations of each part are assigned a priority. Normally a best placement is determined by

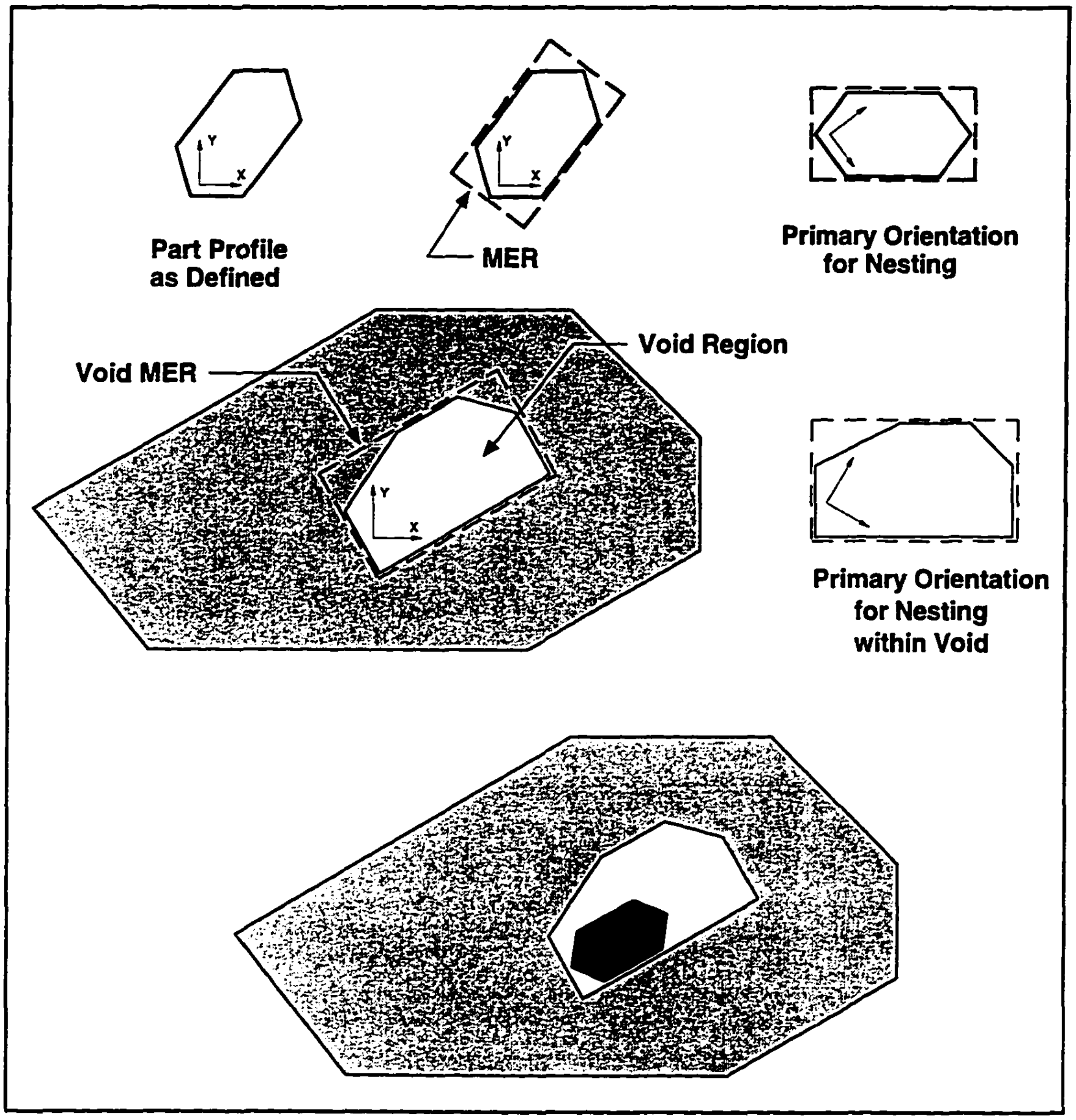


Figure 2.15 The primary orientation for nesting within voids

that orientation causing the leftmost lowest extension of the part, as dictated by the placement policy. In many cases, however, all orientations yield essentially the same positions based on this criterion. In such situations (Figure 2.16), orientation priorities are used.

Preferences are established by calculating the *above* and *right areas* of the part in each of its primary orientations. These areas

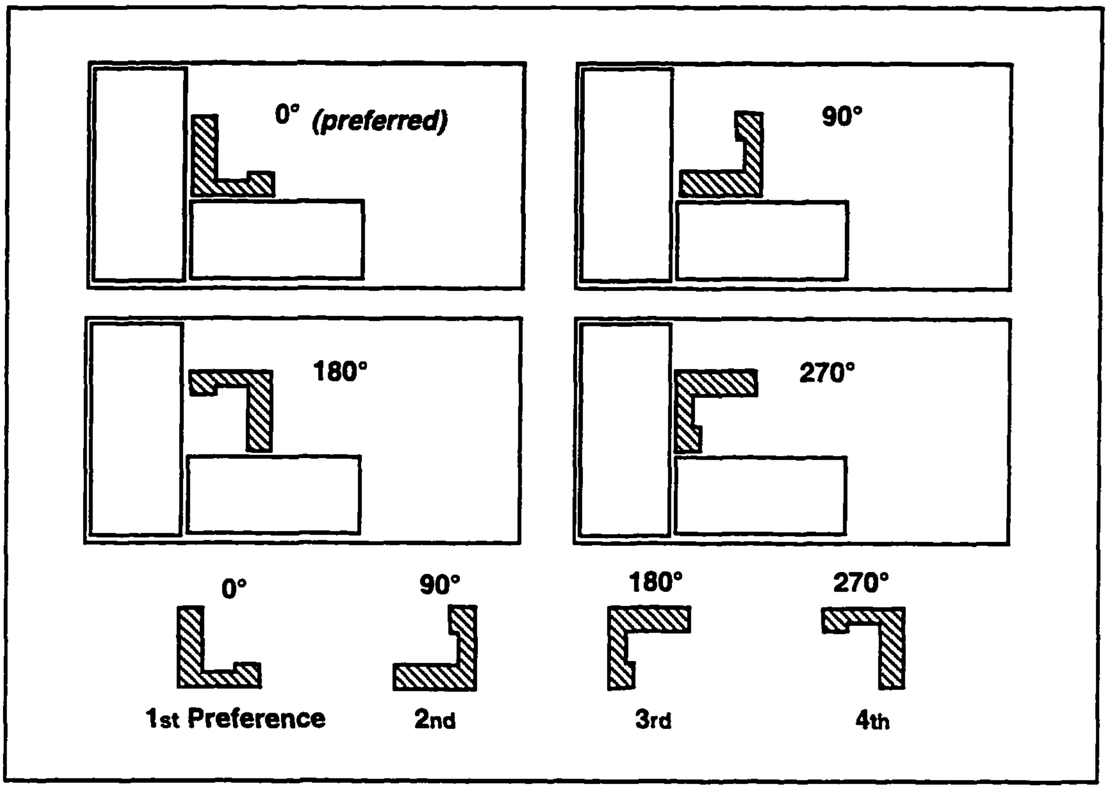


Figure 2.16 Four orientations produce the same leftmost lowest placement. Orientation priorities are used to select the best placement.

may be viewed as rough measures of the shadow cast by a light projecting either from the right or above the part. The right area of a part is found using its rightmost highest and rightmost lowest profile vertices. The part profile is projected horizontally backward from these points until the left most extension of the part is reached, and the area of the enclosed polygon calculated (Figure 2.17). A similar procedure for constructing the above area may be formulated using the highest rightmost and highest leftmost vertices and projecting downward. For a leftmost lowest placement policy, highest priority corresponds to that orientation producing the smallest right area, while second preference is based on the smallest above area (Figure 2.16). These heuristics aid in producing an acceptable placement candidate while limiting the number of NFP's calculated to four for each part.

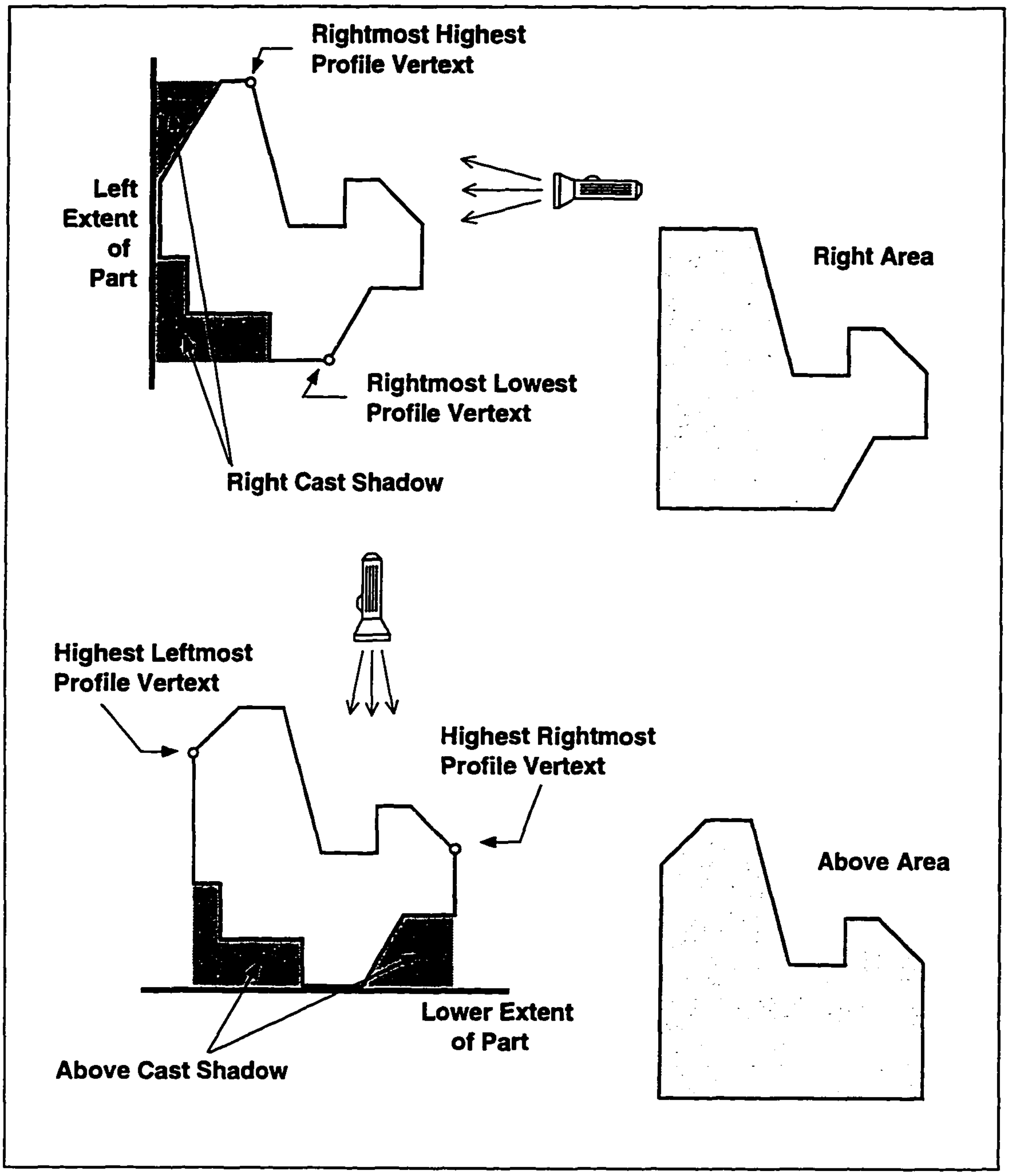


Figure 2.17 Above and left area calculation technique.

2.4 Pilot Study Summary

The solution method described was implemented and tested at an industrial marine fabrication facility. With the exception of input and output file specifications, the package was totally automated. A brief description of the application's basic structure is provided to demonstrate how the various techniques are integrated into a complete solution algorithm. A more detailed discussion of implementation issues and sample nests is deferred until Chapter Five, where results of the primary technique proposed in this manuscript are compared with the methods of this chapter.

Processing begins with the sorting of parts into groups of similar thicknesses and material grade. Prior to nesting each part is simplified and offset by a specified amount to account for the curf of the flame-cutting device. Internal voids are also extracted for possible use as irregular resources. These void areas are nested upon first, using the INFP method. All remaining parts are then allocated to rectangular resources through the traditional NFP technique. Finally, the internal voids and any associated nests are re-inserted onto their parent parts and output files generated.

Results from the NFP/INFP application were acceptable in light of the pilot project objective of producing material estimates for large projects. In addition, the solutions generated also provided initial nests as input to the company's own interactive nesting system. These layouts, however, could regularly be improved through manual modification. Several limitations of the NFP method became evident while attempting to develop a new solution technique capable of improving the results comparable to the manual tweaking.

One major drawback of the NFP technique is the limited number of part orientations available. Due to the computational expense associated with the construction of an NFP, relatively few part orientations can be investigated. Furthermore, the suitability of primary rotations derived from the MER is strongly dependent on the part profile and the nature of the parts already placed. For highly irregular shapes, these orientations are seldom optimal. Similarly, the positions indicated by the unchanging leftmost lowest placement policy are often inappropriate, depending on the layout of current intermediate solutions. Examples of both drawbacks are illustrated in Figure 2.18.

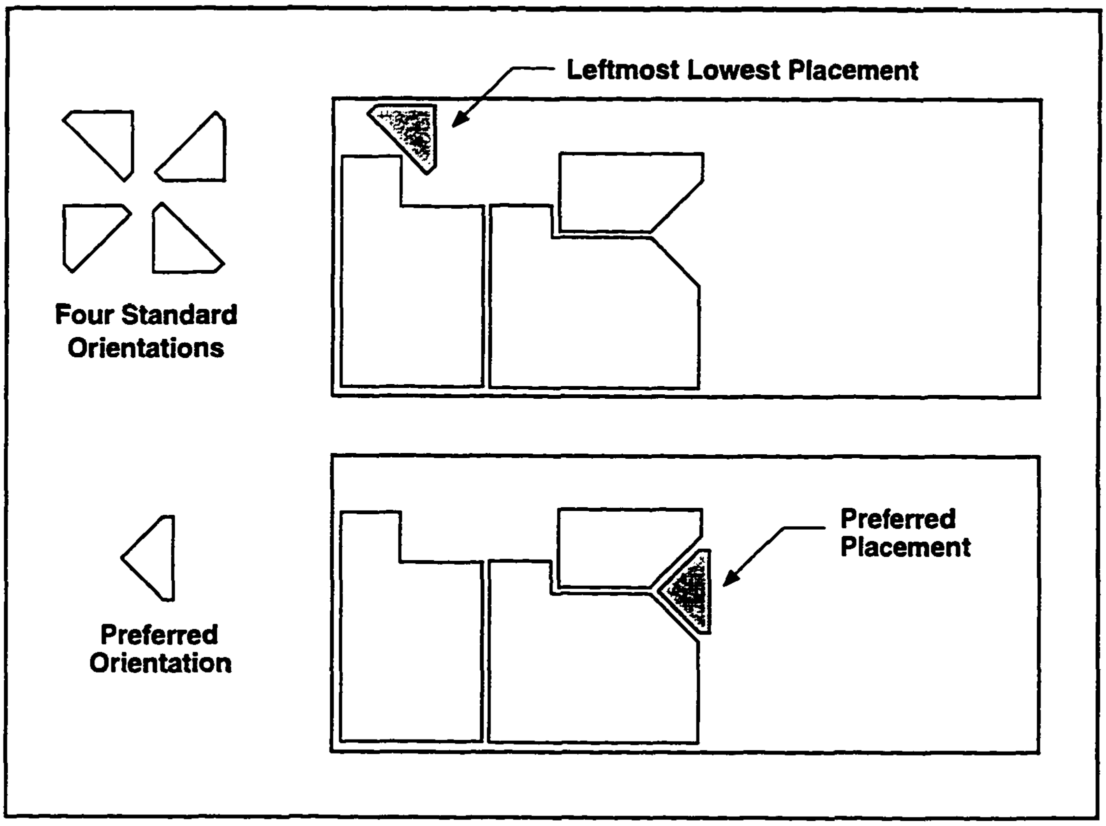


Figure 2.18 Drawbacks of standard orientations and a placement policy

Another concern in implementing the NFP algorithm is its inability to deal with large problems efficiently. At every intermediate stage of the solution, a minimum of one NFP must be calculated for each remaining unplaced part. Over the course of establishing a solution, the total number of NFP's calculated is

$$(N) + (N-1) + (N-2) + \ldots + 2 + 1 = \sum_{i=1}^{N} i \qquad [2.12]$$

where N is the total number of parts to be placed. Furthermore

$$\sum_{i=1}^{N} i = \frac{N^2+N}{2} \quad (N\ even) \quad \frac{N^2-N}{2} \quad (N\ odd) \qquad [2.13]$$

Consequently, the solution cost increases quadratically as a function of the total number of parts. Results from the preliminary investigation confirm this unfavorable non-linear relationship between problem size and computation time. It is unavoidable with the NFP approach.

Each of these issues is addressed to varying extents by the shape reasoning approach proposed in the next chapter.

Chapter Three
Shape Reasoning and the Feature Based Approach

3.1 Introduction

The key principle upon which all shape based techniques are established is the premise that effective nests can be generated using information about the configuration and shape of the previously allocated and unplaced parts of a layout. The general approach of these methods mimics that of human nesters by searching for complementary shapes among the unplaced part profiles and the remaining usable stock material. Reasoning of this type proves useful for accomplishing the two primary tasks of heuristic techniques, part placement and part selection. The most ideal parts for placement may be chosen based on "sameness" of shape, while effective positions and orientations are provided by matching the similar profiles. The success of any such approach is tied directly to the way in which the necessary shape information is represented and extracted. For the solution technique developed, geometric constructs called *features* are used. In this chapter features are defined and their effective use in representing part and resource profiles demonstrated. Chapter Four then details the use of these features in generating intermediate and final solution layouts.

3.2 Features

Before continuing further, some clarification is needed between the common use of "feature" found in technical engineering or

Computer Aided Geometric Design (CAGD) literature and the concept used in this dissertation. The first and most obvious difference exists in dimension. Most CAGD feature applications deal with three dimensional part geometries whereas the current application is limited to purely 2-D profiles. Furthermore, design features are often associated with machine-able entities such as holes and slots. No two dimensional analogy is intended for the profiles. Nonetheless, many of the concepts found in the literature are relevant. In *Parametric and Feature-Based CAD/CAM*, features are defined as "modeling entities that allow commonly used shapes to be characterized and associated with a set of attributes relevant to an application" [SHAH95]. At this abstract level many of the tenets and ideas found in the literature are useful for solving the current problem.

At its simplest, the local shape of a polygon may be described by any pair of adjacent edges, their connecting vertex, and their included angle (Figure 3.1). In this form, a shape may be classified as either concave or convex, based on its associated vertex and the definitions previously noted (§2.2.1). To extend the concept of shape to true protrusions (peninsulas) and pockets (coves) an additional edge is added. This produces the *three sided features* used in this work. In most cases, a sufficient description of shape can be achieved using features of two and three sides. Since exact matches of shape with three or even two sides are uncommon with irregular profiles, little benefit is gained by considering features of four or more sides. In practice, the additional detail is unnecessary and impractical. Experience with manual nesting has shown that, most commonly, pieces are placed in the concave recesses formed by the previously allocated parts. Consequently, it was decided early in the algorithm design process, to limit part features to only convex (peninsular) shapes while remaining resource or void features were restricted to

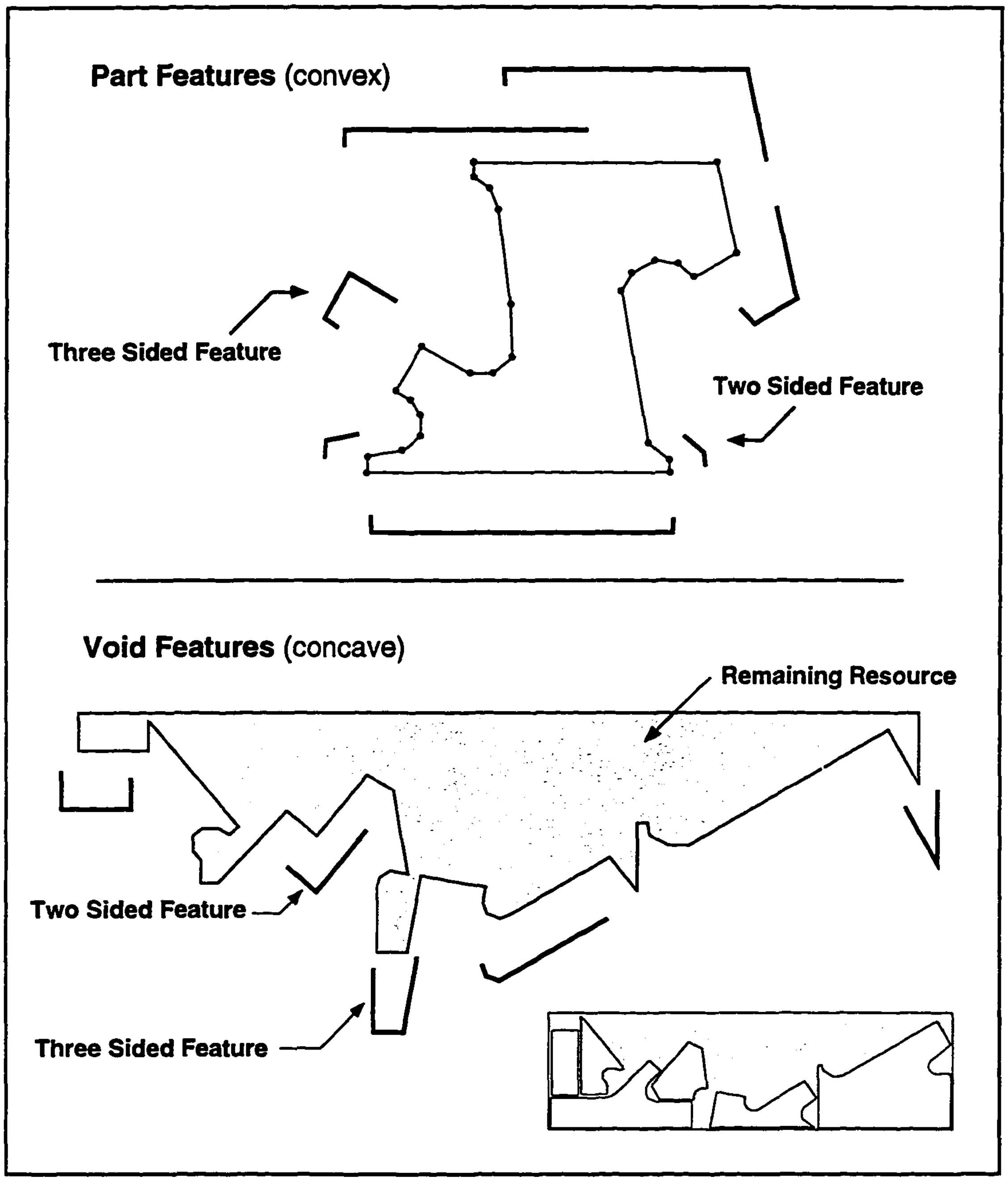


Figure 3.1 Two and three sided features as consecutive edges on part and void (remaining resource) profiles. Part features are convex, while voids are concave.

concave notches. Examples of such void and part features are shown in Figure 3.1.

If restricted to three sequential edges of the actual profile, the notion of a feature still supplies only a localized characterization of the shape. The general problem of nesting parts of varying size and complexity requires information about shape at various scales. When placing small parts finer detail is required. However, this additional detail complicates the positioning of larger parts and is of little use. This is demonstrated in Figure 3.2. To obtain a more global characterization, simplified

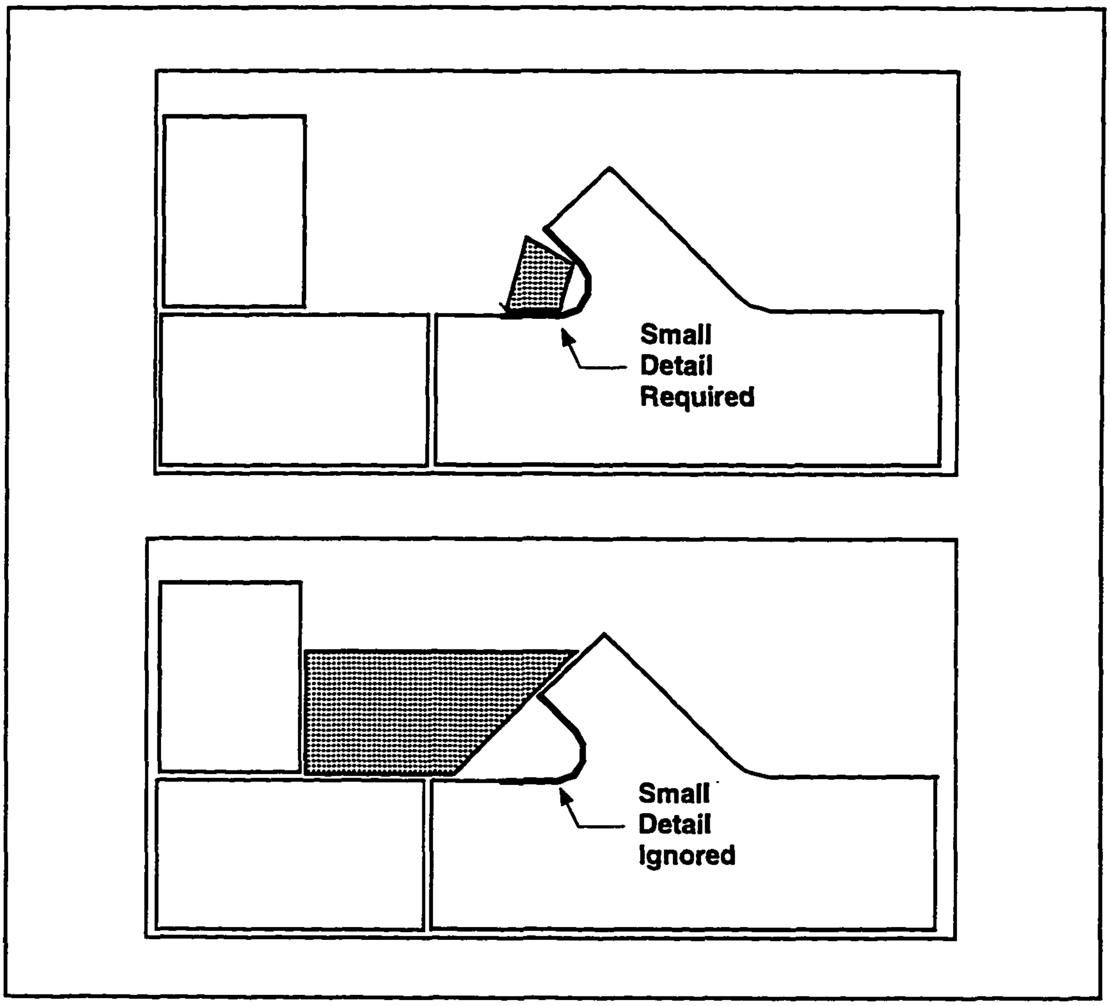


Figure 3.2 The utility of shape characteristics at various levels of detail during part placement.

profiles which approximate the original shape at varying levels of detail are produced. Once such profiles exist, features are easily extracted using straightforward heuristics, which examine each set of three sequential sides and their two included angles. In this way a hierarchical group of features describing both minuscule and large scale characteristics of a profile can be constructed.

Part and void profile simplification is accomplished using a series of techniques detailed in the following sections. The methods for processing are outlined with respect to the remaining resource first, as part profiles are handled using a subset of these same methods. The combination of these techniques to produce the idealized representations needed for effective features is then discussed. Finally the nomenclature and conventions used to define the descriptive information stored with each feature are presented.

3.3 Profile Simplification Techniques

Methods of approximating a part profile with varying degrees of detail have been studied by many [DAVI77], [PAVL74], [PAVL78] [ROSE73], [ROSE75], [HORO75], [FREE74]. The current method proposes to use features for non-overlapping placement, requiring they be extracted from approximations completely inscribing (voids) or circumscibing (parts) the original profile. The need for this restriction will become evident in Chapter Four. This is the major impediment to implementing previously reported techniques. Consequently a series of procedures geared specifically toward the objectives of the allocation problem were developed. These methods are discussed first with respect to simplification of the remaining resource; the same concepts are also applied to parts.

Simplification of the void is accomplished through two general techniques. The first, *small complexity reduction* (SCR), eliminates inconsequential shallows, ridges, and fillets by examining each profile vertex and its two adjacent sides. Since the effect of such methods is localized to only a small portion of the profile, the extent of the simplification is limited. The second technique, *large scale resource division* (LSRD), eliminates complexity at a global level, splitting the void at locations of necking and evaluating the usefulness of the regions based on their shape and size. The remaining resource is simplified when unsuitable areas are eliminated.

3.3.1 Small Complexity Reduction

SCR is divided into three separate routines which eliminate shallows, ridges and chamfers respectively. Acceptable candidates for elimination are detected by calculating the height of each vertex of the void profile. The height of a vertex is the perpendicular distance from the line connecting its two adjacent vertices. This is given as

$$H_i = ((\boldsymbol{E}_i \times \boldsymbol{E}_{CR}) \cdot \boldsymbol{u}_z) \;/\; |\boldsymbol{E}_{CR}| \qquad [3.1]$$

where $\boldsymbol{E}_{CR}$ represents the crossing edge and is defined as

$$\boldsymbol{E}_{CR} = \boldsymbol{V}_{(i+1)} - \boldsymbol{V}_{(i-1)}\ . \qquad [3.2]$$

Negative vertex heights represent concave shallows while positive values represent convex ridges (Figure 3.3). If the magnitude of a vertex height falls below a set tolerance, *SCR_HTOL*, a candidate for elimination exists. The value of SCR_HTOL is selected from experience to be representative of smaller details.

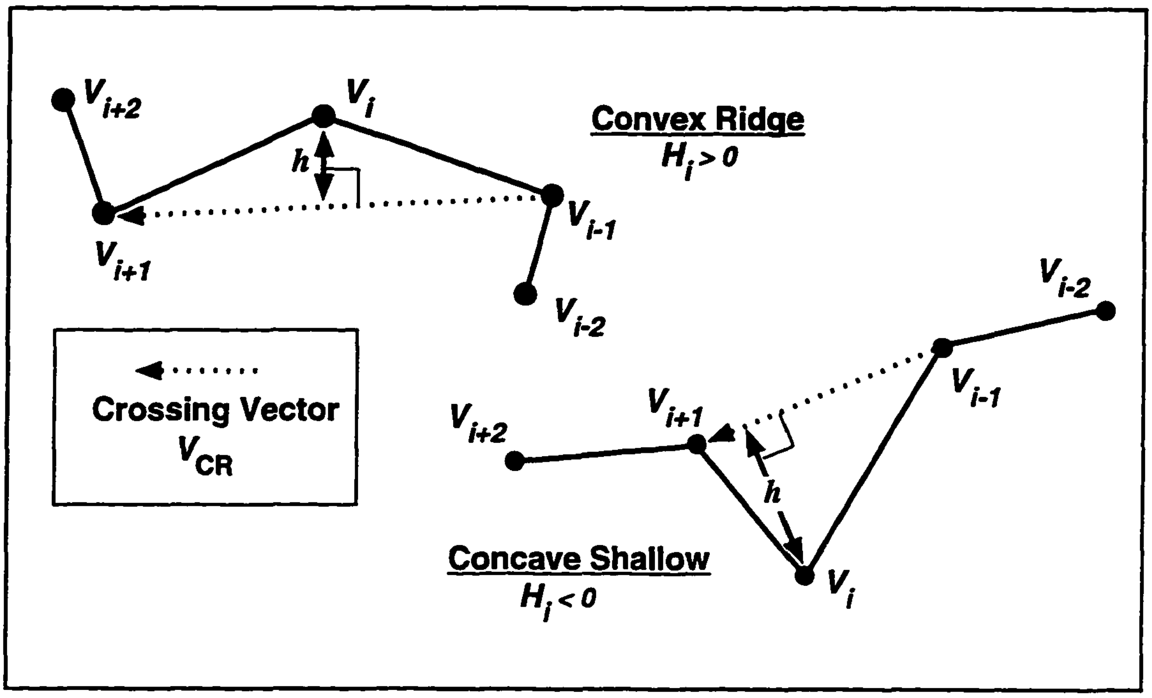


Figure 3.3 A convex ridge and concave shallow and their associated heights

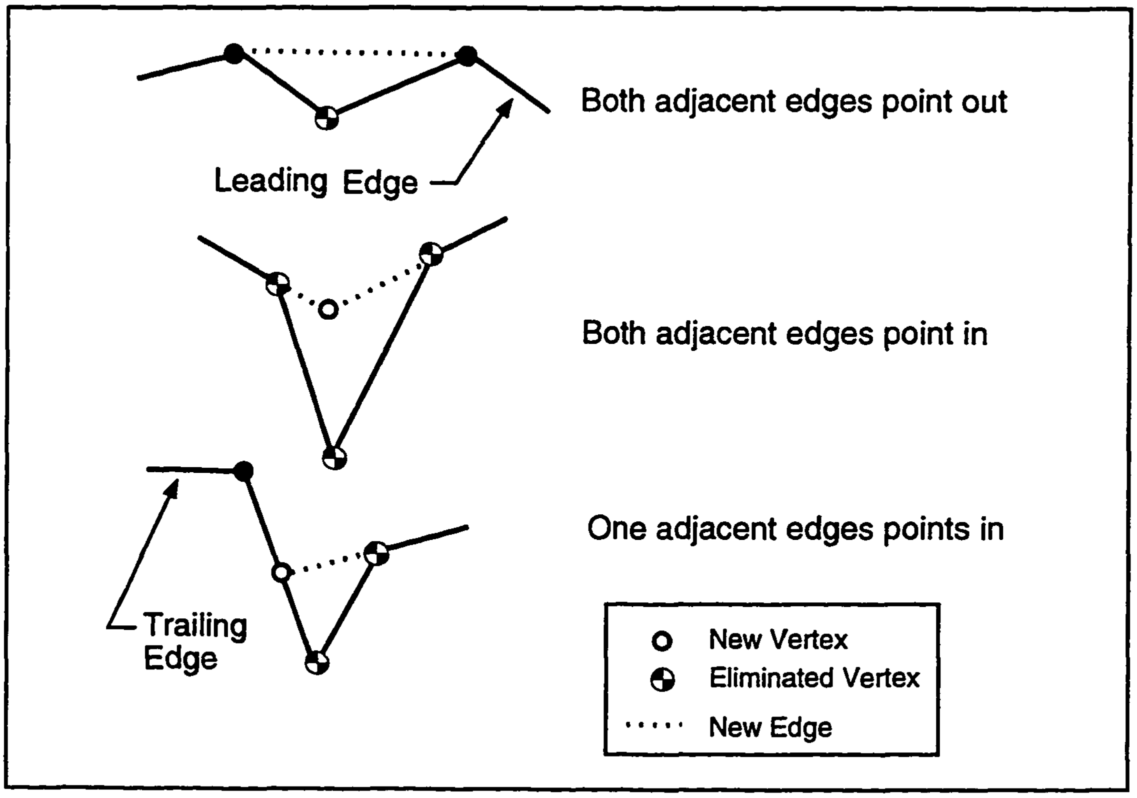


Figure 3.4 Convex shallow elimination methods

Methods for dealing with concave shallows depend upon the nature of the edges immediately preceeding and following the entity. These *adjacent edges* may be classified as either pointing into or out of the shallow. If

$$((\boldsymbol{E}_{(i-2)} \times \boldsymbol{E}_{CR}) \cdot \boldsymbol{u}_z < 0) \;\; AND \;\; ((\boldsymbol{E}_{(i-2)} \times \boldsymbol{E}_{(i-1)}) \cdot \boldsymbol{u}_z > 0) \qquad [3.3]$$

evaluates true, the *leading edge* ($\boldsymbol{E}_{(i-2)}$) is "in", otherwise it is "out". Similarly, the *trailing edge* ($\boldsymbol{E}_{(i+1)}$) is in if

$$((\boldsymbol{E}_{(i+1)} \times \boldsymbol{E}_{CR}) \cdot \boldsymbol{u}_z > 0) \;\; AND \;\; ((\boldsymbol{E}_{(i+1)} \times \boldsymbol{E}_{(i)}) \cdot \boldsymbol{u}_z < 0) \qquad [3.4]$$

evaluates true. The three possible cases and the method of simplification used are shown in Figure 3.4. Each has in common the elimination of at least one vertex and edge.

Convex ridges are eliminated by replacing the two edges forming the irregularity with a single edge parallel to its crossing vector and passing through its defining vertex ($\boldsymbol{V}_i$). Two simplification methods are possible for each adjacent edge of the ridge, depending upon the type of intersection with the new *replacement edge* (Figure 3.5). For the case of the trailing edge, if

$$(\boldsymbol{E}_{CR} \times \boldsymbol{E}_{(i+1)}) \cdot \boldsymbol{u}_z \; < \; 0 \qquad [3.5]$$

is true, any valid intersection point should occur along the length of the existing edge. Otherwise, if

$$(\boldsymbol{E}_i \times \boldsymbol{E}_{(i+1)}) \cdot \boldsymbol{u}_z \; < \; 0 \qquad [3.6]$$

is true, any valid intersection must occur along an extention of the trailing edge. To prevent any undesireable protrusion of this new edge into the void, limitations are placed upon the length of the new trailing edge:

$$|\boldsymbol{E}'_{(i+1)}| \leq |\boldsymbol{E}_{(i+1)}| + 2 \cdot SCR\_HTOL \qquad [3.7]$$

where a prime denotes values after simplification. Both cases and the method of simplification used are shown in Figure 3.5. A similar development can be made for the leading edge of the ridge.

Chamfers are the final class of irregularity dealt with in SCR. Suitability for elimination is based on the height of an edge rather than the height of a vertex. An edge height is determined by extending

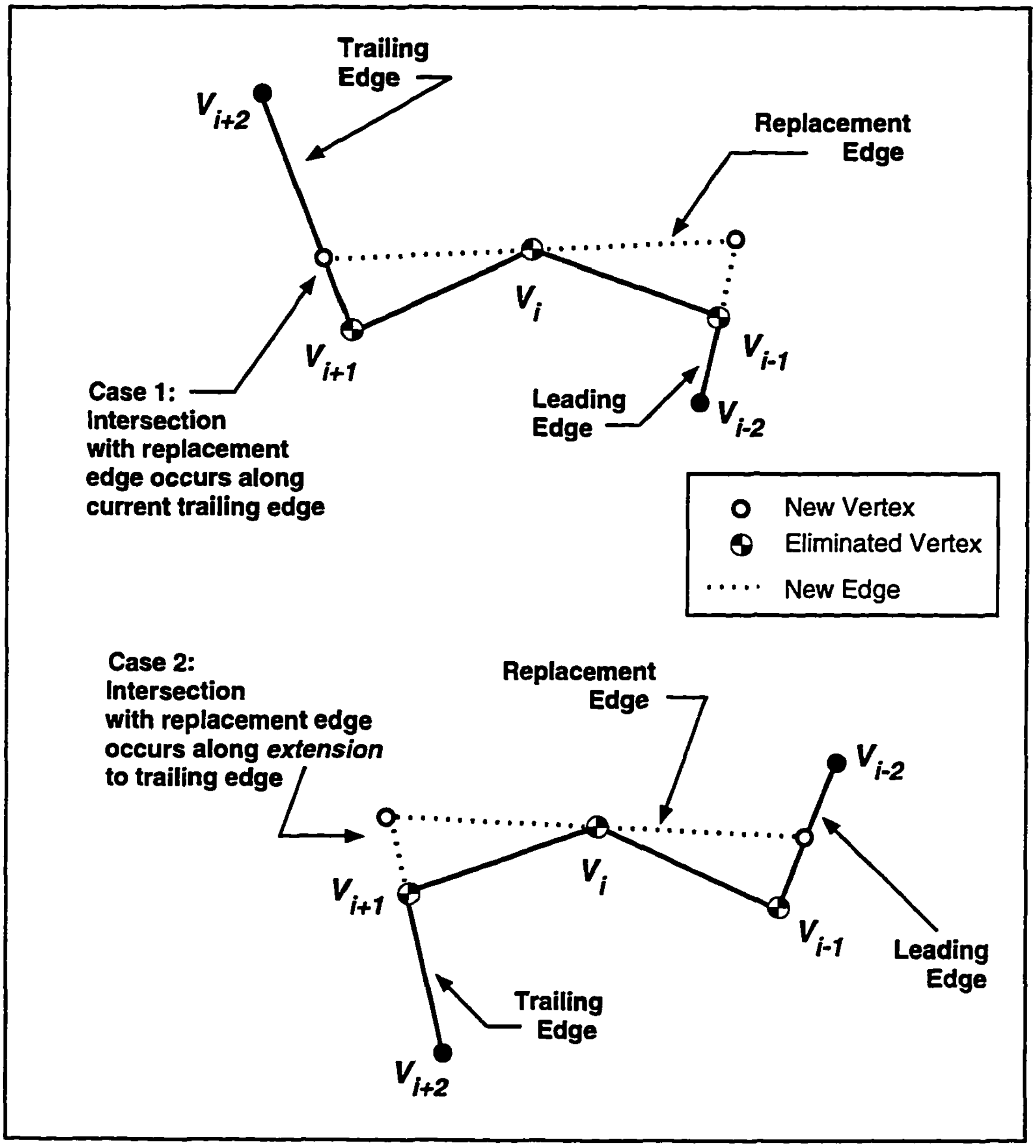


Figure 3.5 Convex ridge simplification showing the two possible cases with the trailing edge.

the two adjacent edges ($E_{(i-1)}$, $E_{(i+1)}$) until an intersection is found. The height of the edge in question is the distance of this intersection from E_i. Only edges defined by two convex vertices need be examined. If this height falls below the SCR_HTOL, a corner is extended as shown in Figure 3.6. As with shallow and ridge elimination, all new edges must be checked for intersection with the current profile.

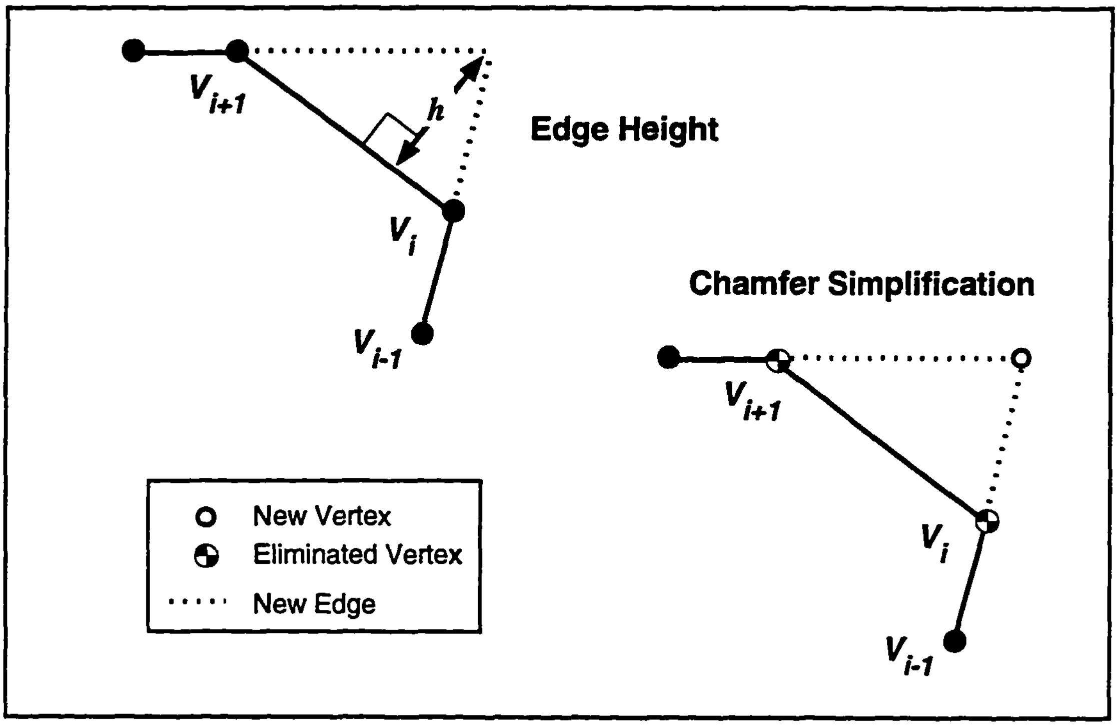


Figure 3.6 Edge heights and chamfer elimination

The elimination methods for each irregularity are implemented in three separate routines : *SHALLOW_RMV( ), RIDGE_RMV( ),* and *CHAMFER_RMV( ).* Although simplification techniques differ, the underlying structure of each routine is the same. Upon entry, the height of all acceptable entities for that routine are calculated once and sorted into ascending order. Irregularities are then removed in sequence from smallest to largest. To prevent potential conflicts with previously removed adjacent irregularities, all alterations and eliminations of edges and vertices are recorded. If any substantive changes have occurred to the profile elements required for the simplification of that entity, it is ignored. Processing continues in this way until all of the non conflicting eligible irregularities are removed.

3.3.2 Large Scale Resource Division

The second method of simplifying the remaining resource is LSRD. LSRD is achieved through splitting a void into several regions at areas where necking occurs. The objective is to maintain larger useful areas, while eliminating smaller unusable ones where the likelihood of successful placement of parts is minimal. Simplification is achieved in multiple steps through the decomposition of the void along *dividing lines* (DL's). The method is depicted graphically in Figure 3.7. At each stage three actions may be taken with each of the available sub-regions.

- DIVIDE Divide the subregion at its shortest DL.
- ELIMINATE Simplify the remaining resource by completely eliminating the subregion.
- ACCEPT End processing of the subregion and incorporate it into the simplified description of the remaining resource.

Four criteria determine the action taken with each subregion. Figure 3.8 indicates how these are used.

- AREA True if the area of the subregion is smaller than a set tolerance (*AREA_TOL*).
- CONCAVE True if all vertices of the subregion are concave.

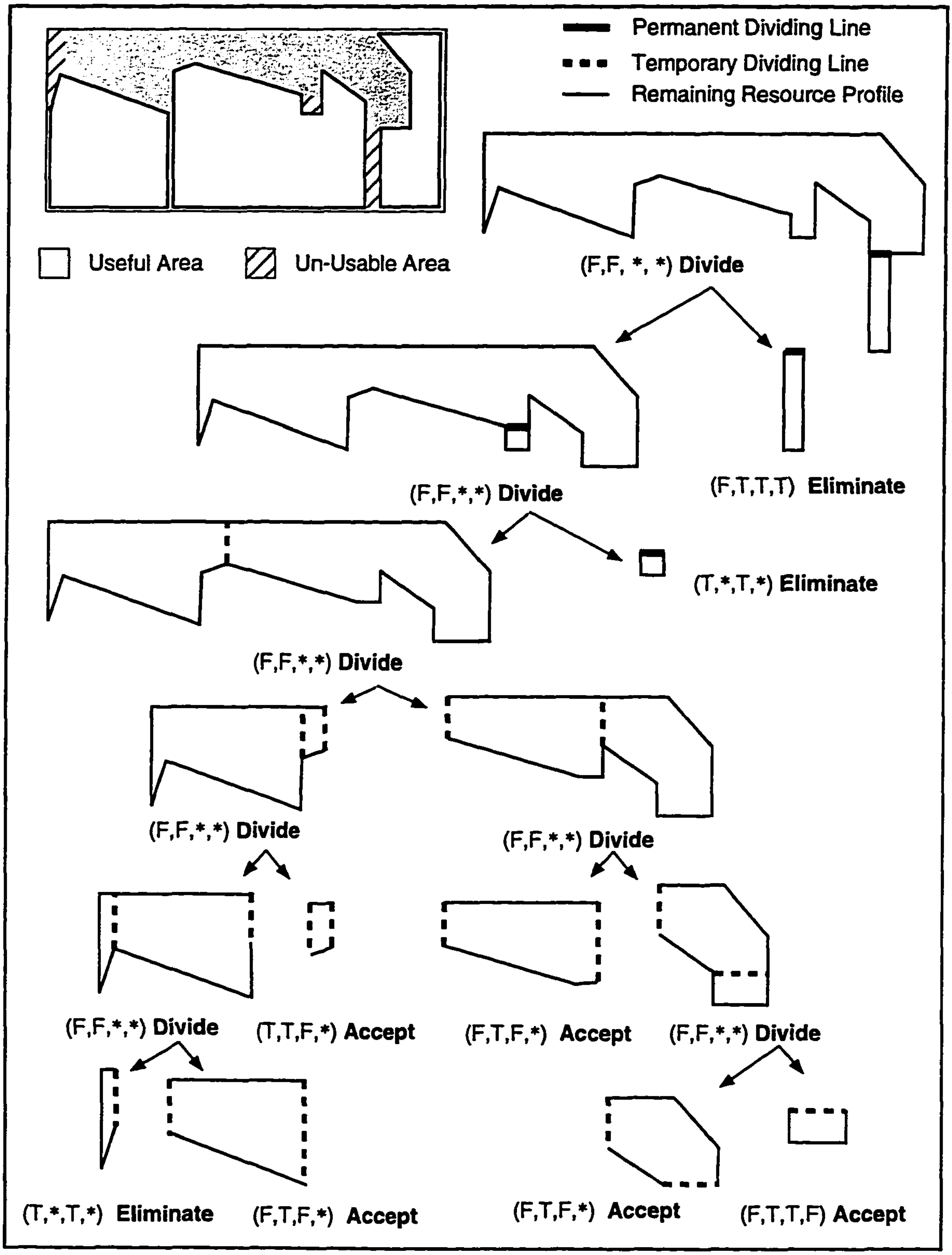


Figure 3.7 The LSRD process. Quadruples preceding each action represent values of the decision criteria (Area, Concave, Side, MER) for that subregion. Asterisks indicate the criteria is not required to determine the action shown.

- SIDE — True if one or none of the edges describing the subregion is a *temporary dividing line.* Dividing lines whose lengths fall below a set tolerance (*LENGTH_TOL*) are considered temporary. All others are referred to as *permanent dividing lines.*
- MER — True if the longest side of the MER for the subregion is smaller than a set length tolerance.

The general intent of LSRD is to eliminate small regions and divide larger ones until only convex areas remain. The existence of a long and narrow areas is determined by examining the MER of the subregion. When appropriate these are also eliminated. Due to the nature of decompostion, small centrally located regions may also be produced. By limiting the number of temporary dividing lines used to describe the boundary

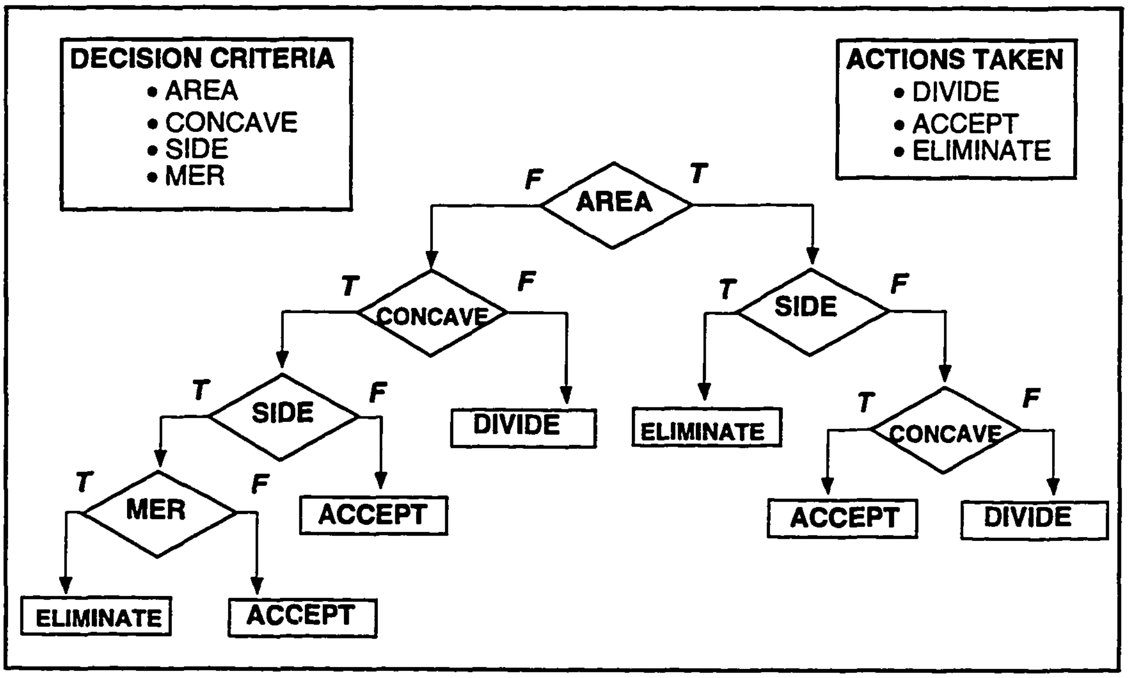


Figure 3.8 Decision tree showing the action taken for void subregion criteria

of the of subregions, their unnecessary elimination is prevented. Examples of each condition can be found in the idealized profile of Figure 3.7.

The degree of simplification produced by LSRD is determined by the values of *AREA_TOL* and *LENGTH_TOL*, which in practice are adjusted to reflect the representative size and dimension of the set of remaining parts. To achieve this goal, each profile's characteristic dimensions are represented by the height and width of its MER, while its area is used to indicate size. Parts previously placed and those larger than the current resource are eliminated. The distribution of these measures across the part set is highly variable and unknown [HART85], [SUMM87]. Consequently, an approximate description is generated by sorting the remaining values, and tabulating the averages for the smallest, middle and largest third. The three tolerance pairs are generated to produce the different levels of detail needed for nesting. Results achieved by varying the two parameters are shown in Figure 3.9. After simplification what remains are the useful regions suitable for the profiles described by a tolerance pair.

The LSRD technique is implemented in a routine called *DIVIDE_REGION( )*. The algorithm is structured to divide and simplify the profile it receives until a permanent dividing line is encountered. When this occurs, processing stops and the two separate subregions produced are returned, along with any simplifications to their profiles accomplished prior to this division. Otherwise processing continues until no further simplifications can be made, and the resulting *totally reduced* profile is output. Structuring of this type allows integration with the previously mentioned SCR routines.

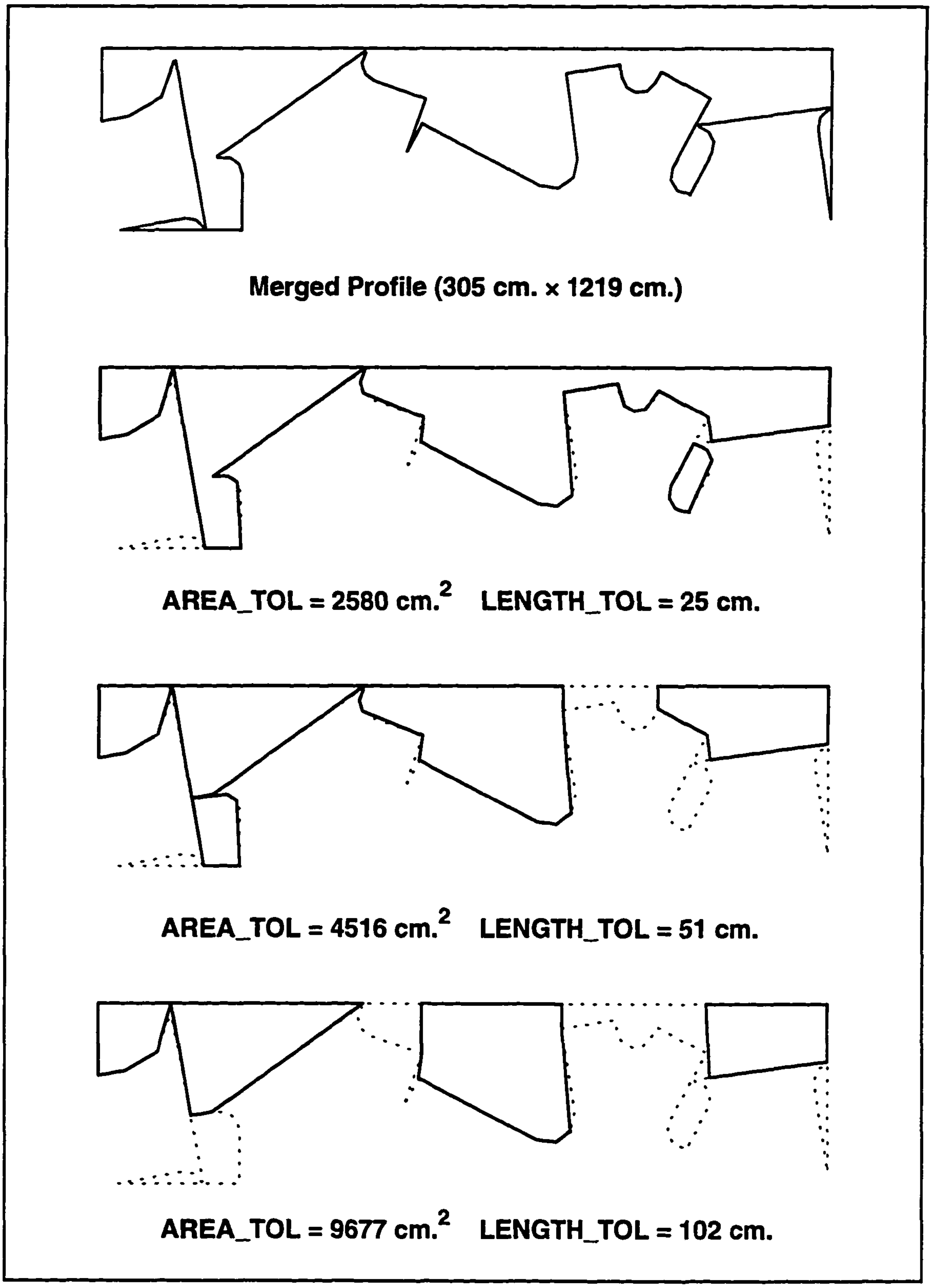


Figure 3.9 Different levels of detail produced by varying the area and division tolerances of the void simplification algorithm.

3.4 Void Simplification

The SCR and LSRD techniques are combined, as shown in Figure 3.10, to produce the profiles from which void features are extracted. Input to the algorithm is supplied by a queue of profiles, which

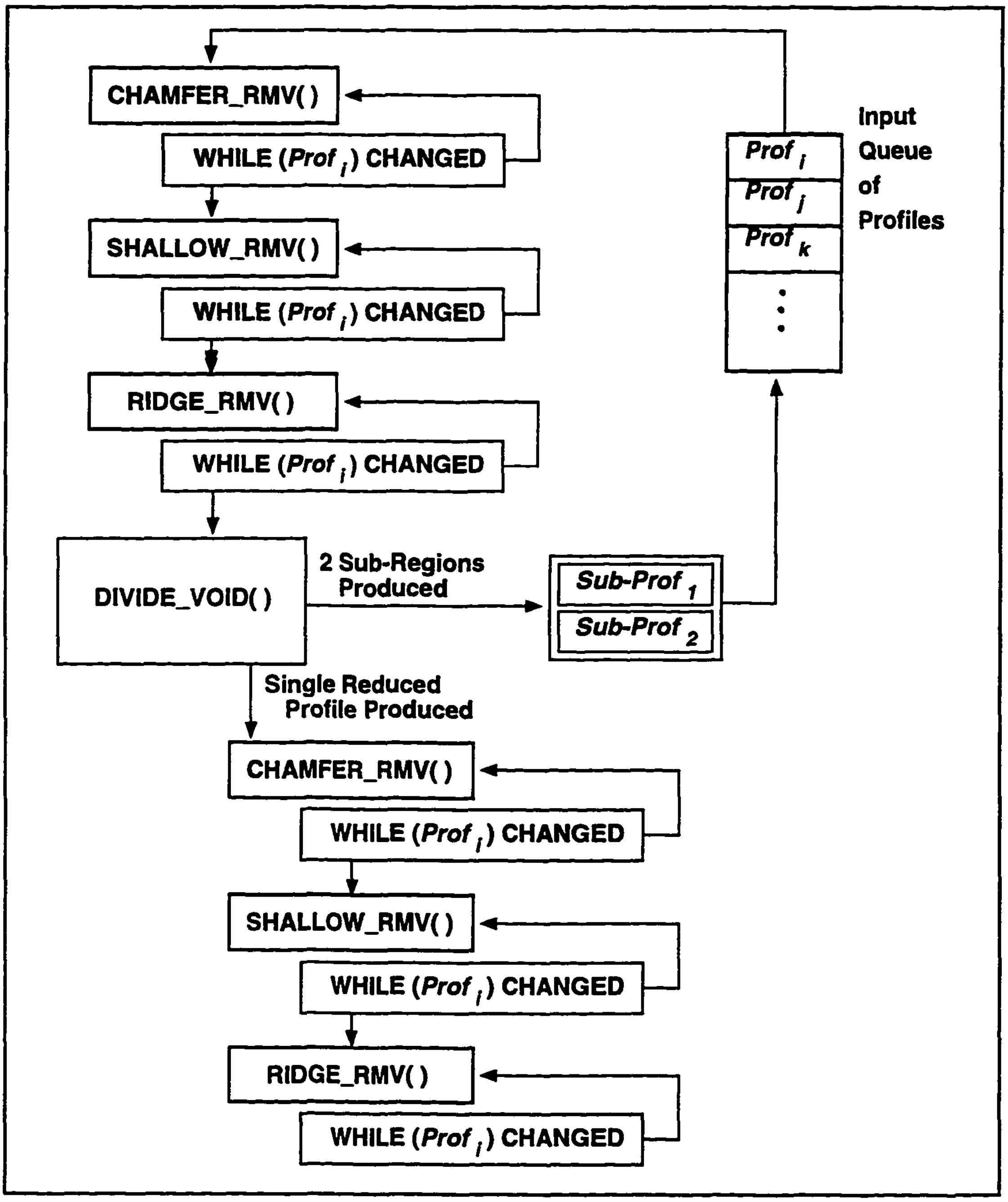


Figure 3.10 The void profile simplification algorithm

initially contains only the unsimplified remaining resource. Each profile is originally processed using the SCR routines. The CHAMFER_RMV( ) routine is used first, and is called repeatedly until no further simplifications can be made. The concave shallow and convex ridge routines follow in a similar fashion. The resulting profile is then operated on by the DIVIDE_VOID( ) algorithm. If two subregions are produced, each is submitted to the input queue for further processing. When a single reduced profile results, it is subjected to the SCR routines one final time and the output stored. Processing continues in this way until the input queue is emptied. As shown in Figure 3.9 multiple disjoint regions can result.

3.5 Part Simplification

Unlike the remaining resource, approximations of the part profiles are produced using only SCR methods. Several arrangements were investigated, however the combination of routines shown in Figure 3.11 produced the best results over a wide range of shapes. Initially, the part profile is simplified by executing SHALLOW_RMV( ) repeatedly until no changes can be made. This is followed by a single run with the chamfer and ridge removal algorithms. The entire sequence is then repeated until no unacceptable irregularities remain. Differing levels of detail are achieved by varying the maximum allowable irregularity height, SCR_HTOL. Since this differs depending on the size of the part, the tolerance is set dynamically as a percentage of the profile's MER diagonal. For the current application, 15%, 20% and 30% are used to produce profiles of small, medium, and large detail. Sample results are shown in Figure 3.12.

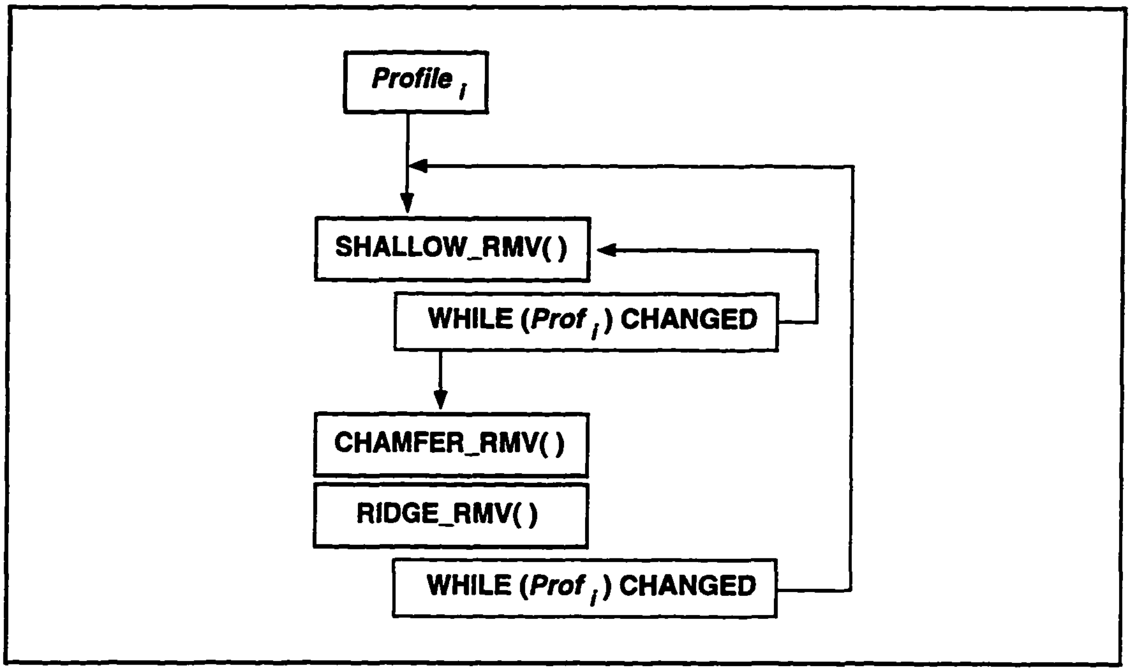


Figure 3.11 The part profile simplification algorithm

3.6 Feature Extraction and Storage

To produce a descriptive set of features for a void, its profile is first approximated at three levels of detail through profile simplification. A similar procedure is followed for parts, with the MER also added to the normal collection of approximating profiles. Features are easily determined by searching the descriptions generated for the required cove and peninsula shapes. For voids, a *three sided feature* coincides with each pair of adjacent concave vertices, while for parts a pair of convex vertices is needed. Other useful sections of the remaining resource are detected by permitting a special *two sided void feature.* As shown in Figure 3.13, these areas coincide with a single concave vertex bordered by a convex vertex on both sides. With this exception, all other part and void features are three sided.

A separate set of features is extracted from each representation generated through profile simplification. Although these profiles

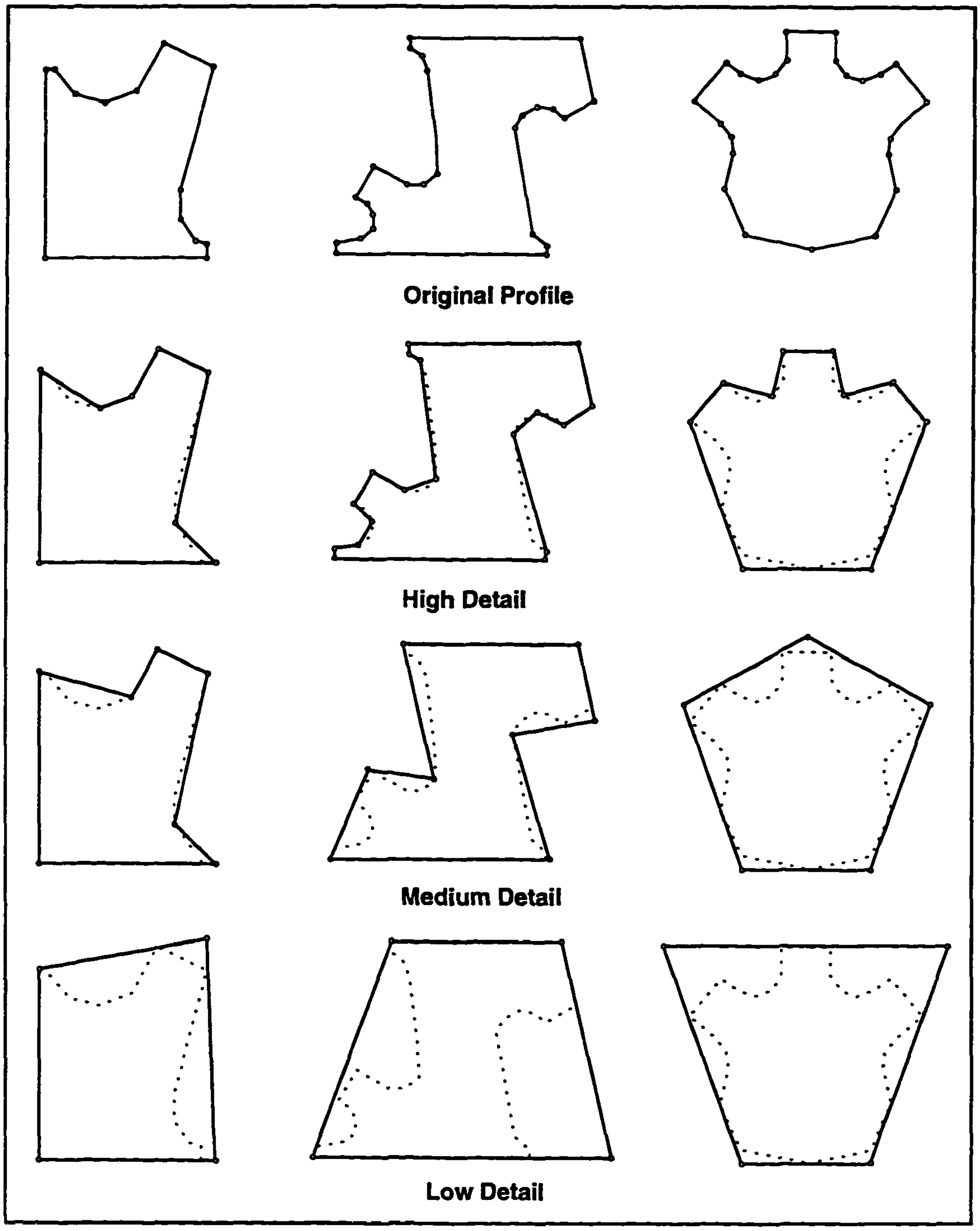


Figure 3.12 Different levels of detail produced by the part profile simplification algorithm

correspond to different levels of detail, identical features can occur and are eliminated. Those remaining are then combined to produce a single set of unique features for each part or void.

Information used for the selection and placement of parts is stored with each feature. With reference to the remaining resource, this

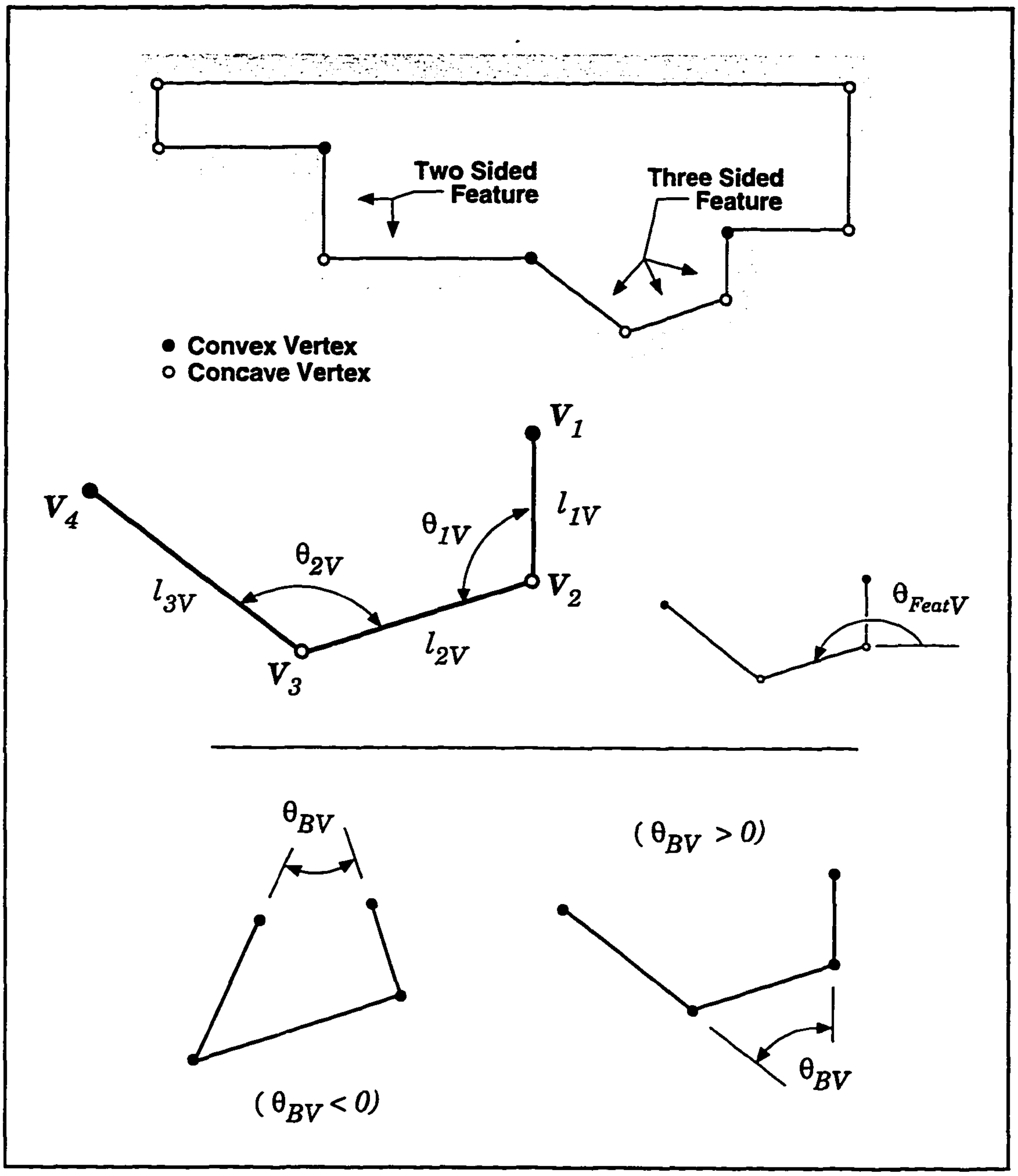


Figure 3.13 Information stored with each feature

includes the length of each side (l_{1V} , l_{2V} , l_{3V}), and the two included angles (θ_{1V} , θ_{2V}). Numerical labeling indicates the order of appearance in the clockwise oriented void profile. The angle between sides one and three of the feature is calculated (θ_{BV}), giving an indication of the "open" ($\theta_{BV} > 0$) or "closed" ($\theta_{BV} > 0$) nature of the profile segment (Figure 3.13). The void feature's position with respect to the original (unsimplified) remaining resource is maintained by storing its constituent vertices (V_{1V} V_{2V} , V_{3V} , V_{4V}) as measured in the stock plates local coordinate system. Vertices 2 and 3 are referred to as *interior*, while 1 and 4 are *exterior*. The feature orientation (θ_{FeatV}) is specified by the angle between the horizontal and the *middle*, *center*, or second side (V_2V_3) of the feature. Two sided features are treated as a special case of the three sided variety, with either l_{1V} and θ_{1V}, or l_{3V} and θ_{2V} set to zero, and θ_{BV} left undefined.

Similar information is stored with each part feature using the same notation and a subscript "P"; however, the part's local coordinate system is used. Although part profiles are oriented counterclockwise, their feature information is labeled in a clockwise fashion for consistency when doing void feature comparisons.

With features constructed and stored, the basic information required for a shape reasoning solution is available. As will be shown in the next chapter, this information can be exploited to both generate and select candidate part placements.

Chapter Four
Nesting with Features

4.1 Introduction

Building layouts through the sequential addition of parts to a resource involves two primary tasks, part selection and part placement. Each stage of the solution requires that a choice be made from the infinite number of locations and orientations possible for each of the unplaced parts. The geometric information of the part and void features is exploited to reduce the options from an infinite number to a workable set.

The use of features for the selection and placement of parts is detailed in this chapter. Although selection would seem to logically precede placement, in practice the decision strategies for both are intertwined. Several precepts are common to both processes and are best understood in the context of part placement. With this in mind, placement is detailed first.

4.2 Part Placement

Placement of a part onto the resource requires establishing both its orientation (θ) and location (x,y). By "matching" complementary void and part features key information is provided for determining both.

Positioning is achieved through a series of steps, with a part orientation selected first. A desirable value is picked from those rotations aligning the sides of the paired part and void feature. The part orientation is then held fixed and its best location found.

The initial part position is establish by determining the way in which the two involved features mesh or fit together. Each of the different possible fits is referred to as an *align type*, and criteria determining the existing case may be derived. The objective of these arrangements is to provide an initial non-overlapping placement of the part which is immediately adjacent to a complementary void shape. Since features are only an approximate representation of the actual void and part profiles, the placement indicated by the align type is only an initial estimate. If the part falls totally within the void, then its position is further refined by shifting along the true void boundary until a final placement is found. Each of these steps is detailed further in the following sections.

4.2.1 Orientation

Potential part orientations are derived by matching features. For a given pair, the developed method supplies three orientations corresponding to alignment of each part feature side with its complement on the void (Figure 4.1). For alignment of side two of the part feature with side two of the void feature, the required rotation (θ_{ROT}) of the part is given as

$$\theta_{ROT} = \theta_{FeatV} - \theta_{FeatP} \ . \qquad [4.1]$$

Similarly, for side one alignment the rotation angle is

$$\theta_{ROT} = \theta_{FeatV} - \theta_{1V} - \theta_{FeatP} + \theta_{1P} \ , \qquad [4.2]$$

and for side three

$$\theta_{ROT} = \theta_{FeatV} + \theta_{2V} - \theta_{FeatP} - \theta_{1P} \ . \qquad [4.3]$$

The aligned edges in each case are referred to as the *primary sides* of the match.

Part rotations for matches between two sided void features and three sided part features are also provided through side alignment.

Four potential orientations are allowed (Figure 4.2). In order to achieve the desired rotations while retaining the convention of "like" side alignment, each two sided feature is stored twice in the conventional three sided format; however one edge is left null. For the first representation side three is omitted while in the second side one is absent (§3.6). In this way, the required permutations may be generated by aligning the available corresponding part and void feature sides.

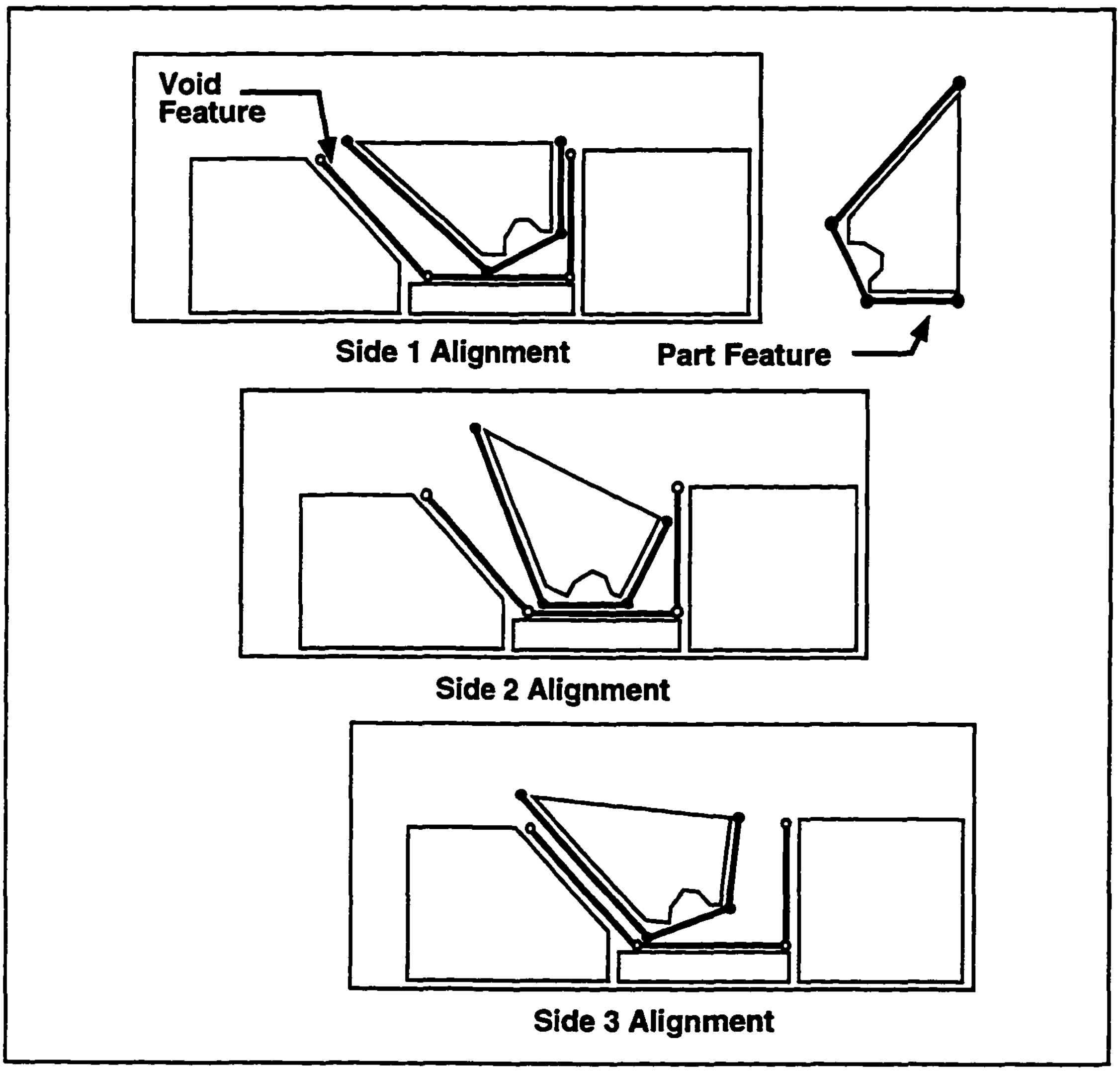


Figure 4.1 Potential feature - void alignments for three sided features

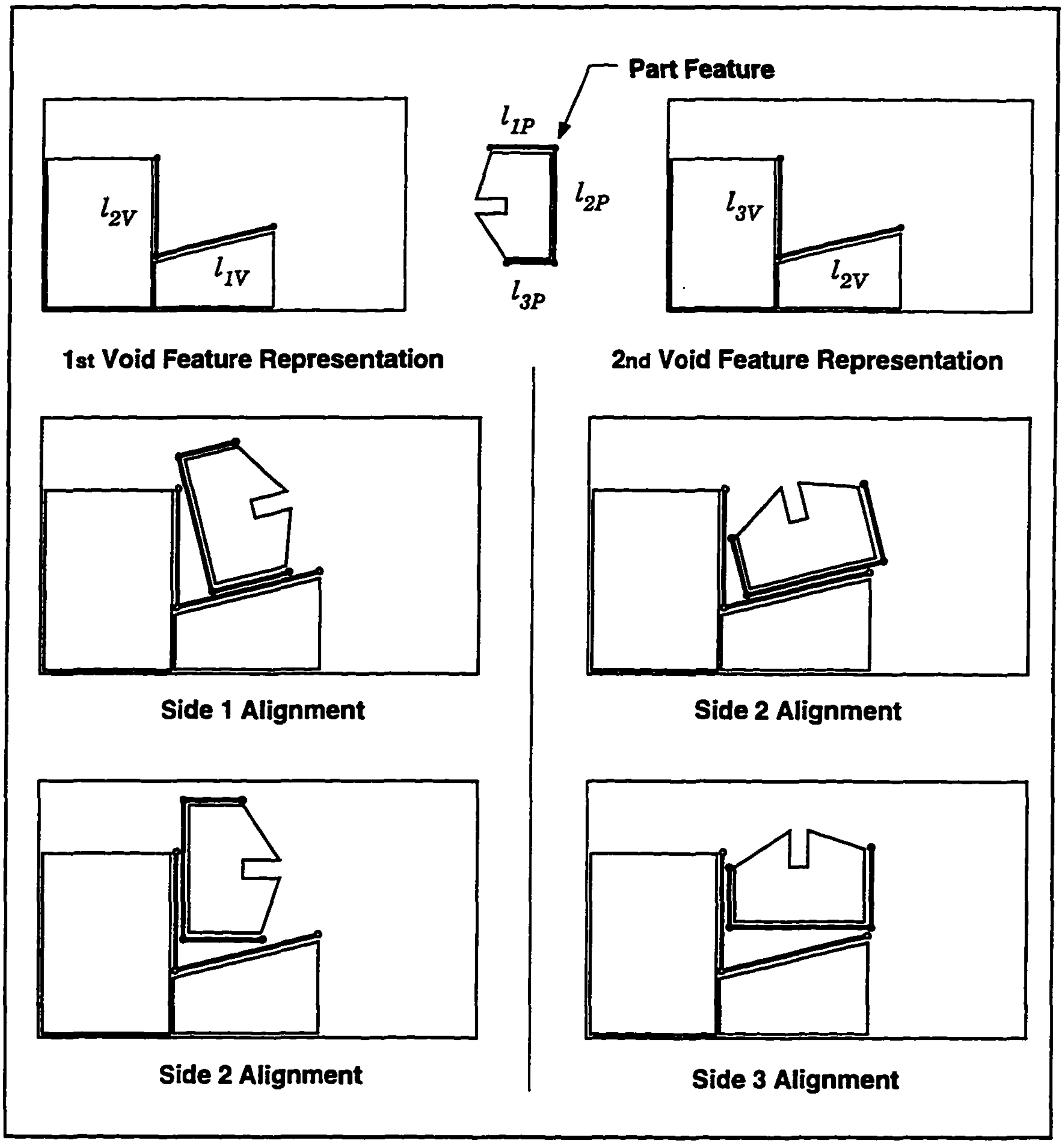


Figure 4.2 The four orientations provided for 2 sided void features

4.2.2 Align Types

Once the primary side of a match is selected, an *align type* or the way in which the two features mesh or fit together is established. Figure 4.3 shows a few of the many align types possible for each of the primary side alignments. A complete list of all supported types is provided in the Appendix A.1. Resolution of the align type determines the relative location of the part feature with respect to its void complement. Since part orientation (θ_{ROT}) and the location of each feature with respect to the entity it describes are known, finding the relative positioning of the two provides the remaining information necessary to initially place a part (x,y).

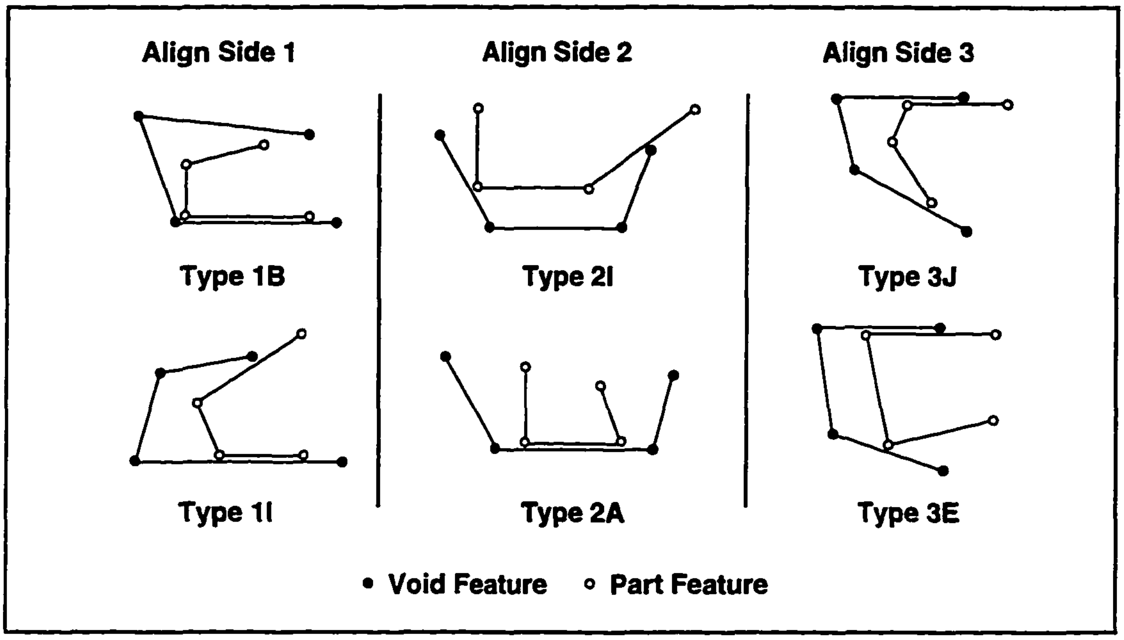


Figure 4.3 Different fits or align types possible between two features

The arrangement of each align type is chosen such that the part feature is shifted as far as possible into the void feature. The objective of such positioning is to provide an initial placement of the part totally contained within the remaining resource and adjacent to a complementary shape. These align type positions are characterized by finding a point of contact between the void and part features. Different interactions may occur, but all can be represented as either a part feature vertex falling upon a void feature side, or a void feature vertex falling upon a part feature side. Regardless of which case exists, these intersecting entities are referred to as the *align type reference vertex* and the *align type* *reference edge,* respectively. The location of the reference vertex upon the reference edge is designated as the *align type reference distance* (s_{AT}). The correct elements for each align type are listed in Tables A.1 through A.3.

For any match of two features, a series of calculations can be performed to determine their relative positions. Although each match is unique, commonalties between cases can be exploited to simplify the procedure. Each align type represents a group of matches having the same conditions for existence. In all cases these criteria can be expressed as a system of linear equations and a set of inequalities constructed from the feature data itself. Satisfaction of the criteria also determines the relative position of the features themselves. This information is catalogued for all supported types. Thus, the arrangement of a particular match may be determined by comparisons to the required criteria stored for each align type.

The set of supported align types are further organized into two large groups or *classes*, based on similarities in their existence criteria. The *touching class* contains all cases where one or both interior vertices of the part feature contact the center edge of the void feature. The second *non-touching class* contains all others. Physical similarities between all matches in a class permit the analogous formulation of the inequalities and equations needed for their existence. An example from each class is described below. The defining criteria for all other align types within each class may be derived using similar reasoning.

Align type 2C (Figure 4.4) represents the most general case of the touching class. Shifting along the primary sides is possible once the part feature has been translated its farthest extent into the void. This extra degree of freedom is eliminated by shifting side two of the part feature as far as possible toward one of the non aligned sides of the void. The direction chosen is based on the absolute difference between the corresponding angles of the two features. If

$$|\theta_{1V} - \theta_{1P}| < |\theta_{2V} - \theta_{2P}| \qquad [4.4]$$

is true, shifting is toward l_{1V}, otherwise it is toward l_{2V}. The edge selected is designated the *secondary side* of the match, as it is the edge most closely associated with the primary sides. Although variable when aligning the middle side of two features, for alignment of sides one or three it is always side two.

Regardless of the side chosen as secondary, the farthest extent which the part may be moved in either direction must be calculated to determine if type 2C exists. With reference to side one, two cases may occur. The first edge of the part feature may be either "in" or "out" of the void (Figure 4.4). Using the triangle formed by l_{1V}, l_{2V}, and l_{1P}, it can be shown that if

$$l_{1P} \cdot sin(\theta_{1P}) \;/\; sin(\theta_{1V}) < l_{1V} \qquad [4.5]$$

is true, l_{1P} is "in" and the location of the reference vertex (V_{1P}) along the reference edge is given as

$$(s_{AT})_{SIDE\,1} = sin(\theta_{1P} - \theta_{1V}) \cdot l_{1P} \;/\; sin(\theta_{1V}) . \qquad [4.6]$$

Otherwise side one is "out" and

$$(s_{AT})_{SIDE\,1} = sin(\theta_{1P} - \theta_{1V}) \cdot l_{1V} \;/\; sin(\theta_{1P}) . \qquad [4.7]$$

A similar formulation can be made for side 3. Case 2C exists if

$$(\,(s_{AT})_{SIDE\,1} + (s_{AT})_{SIDE\,3} + l_{2P}) \le l_{2V} . \qquad [4.8]$$

Triangular constructs such as the one used above may be formed for all remaining align types in the touching class.

Figure 4.5 shows a representative example from the non-touching class, align type 2F. Verification of this type and the relative position of each feature within it may be determined by examining the vector loop formed by vertices V_{1V}, V_{2V}, V_{3V}, V_{4P}, V_{3P}, and V_{2P}. All lengths and orientations within the circuit are known, excepting distances along side three of the void and side one of the part. These variables are represented as l'_{3V} and l'_{1P} respectively. A solvable system of linear equations in the two unknowns may be constructed by summing

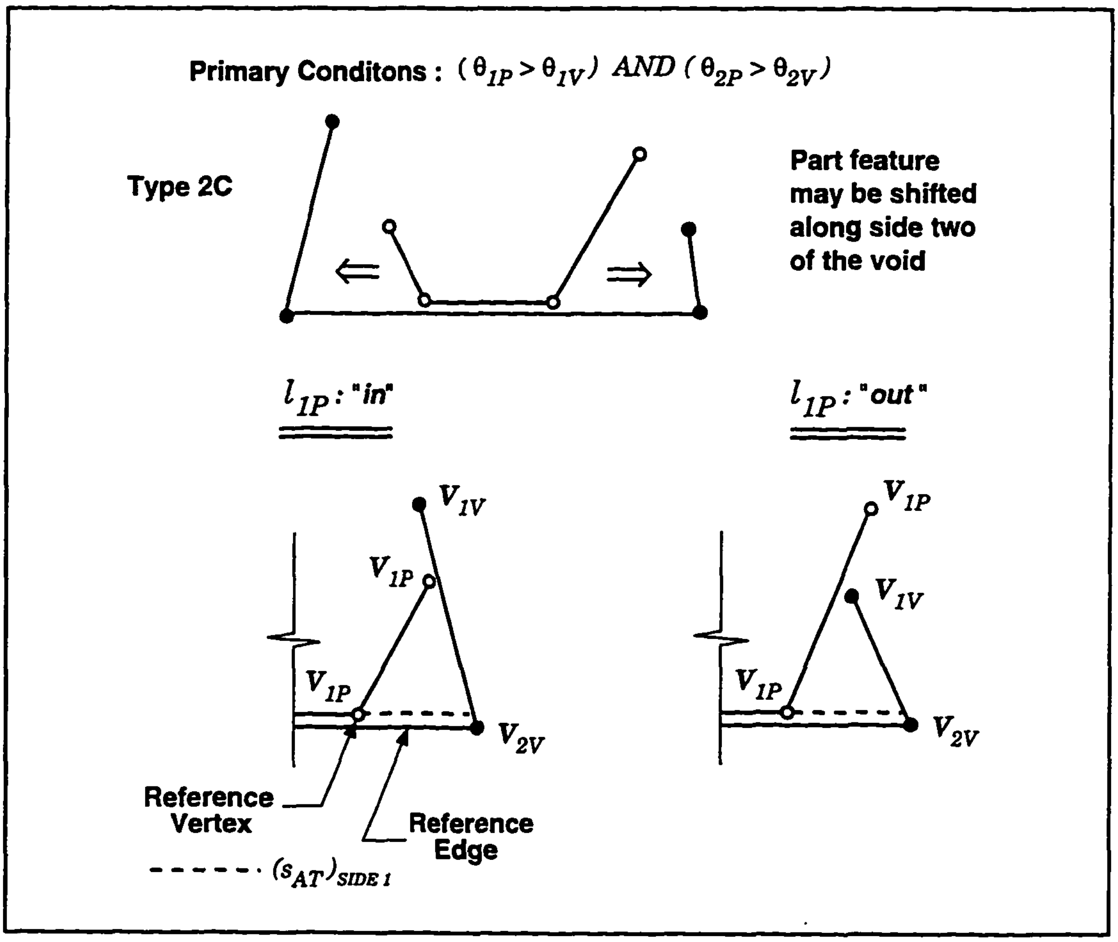


Figure 4.4 A touching class align type (2C) showing the reference vertex, edge and distance (s_{AT}).

distances around the loop in the horizontal and vertical directions. The following result is derived.

$$l_{2V} - l_{1V} \cdot \cos(\theta_{1V}) + l_{3P} \cdot \cos(\theta_{2P}) - l_{2P}$$
$$= l'_{3V} \cdot \cos(\theta_{2V}) - l'_{1P} \cdot \cos(\theta_{1P}) \quad [4.9]$$
$$l_{3P} \cdot \sin(\theta_{2P}) - l_{1V} \cdot \sin(\theta_{1V})$$
$$= l'_{3V} \cdot \sin(\theta_{2V}) - l'_{1P} \cdot \sin(\theta_{1P}) \quad [4.10]$$

Using these equations, the values of l'_{3V} and l'_{1P} may be calculated. The following metric is then used to verify the type. If

$$(0 < l'_{3V} \leq l_{3V}) \text{ AND } (0 < l'_{1P} \leq l_{3P}) \quad [4.11]$$

then type 2F exists. The relative location of the part is provided by setting s_{AT} equal to l'_{3V}.

All non-touching align types contain a vector loop which may be formulated in terms of two unknown edge lengths. By constructing the necessary loop equations for each arrangement in advance, a simple and quick method for determining align type is available for all arrangements in this class.

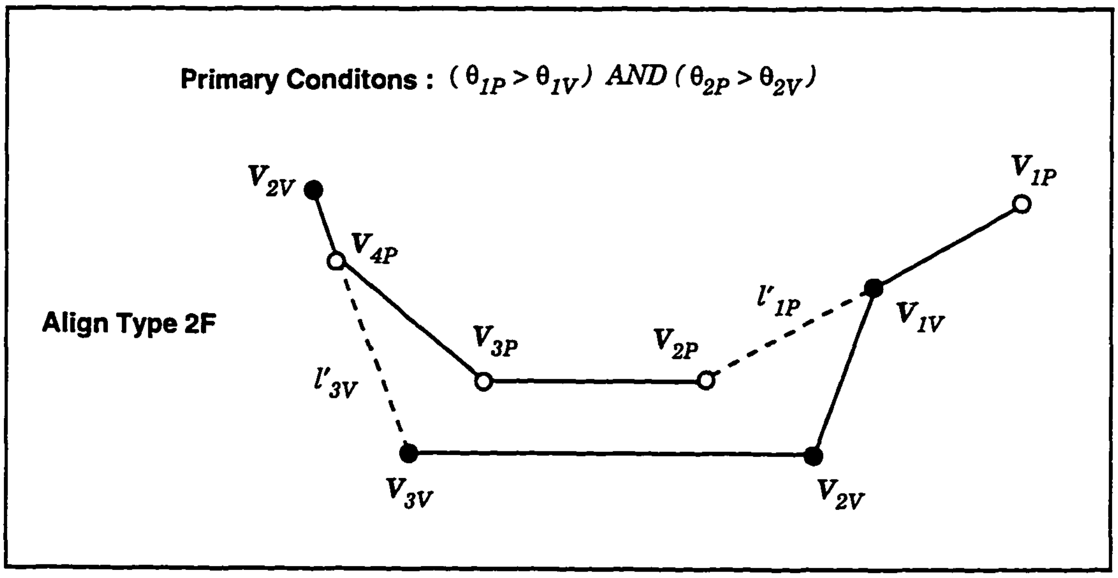


Figure 4.5 Non-touching class align type 2F

4.2.3 Initial Placement and Shifting

Using the information provided by the align type and primary sides, an *initial placement* of the part onto the resource may be generated. This placement is only an estimate of the part's final location however, as the void and part features used for its generation are only approximate representations of the actual profiles involved. A final placement is produced by sliding the part along the actual boundary of the remaining resource using the INFP technique discussed in chapter 2. To guarantee construction of the INFP, the initial placement must be *valid* (i.e. the part must be totally contained within the void). Conversely, an *invalid initial placement* occurs when any portion of the part profile falls outside of the remaining resource, usually at some distance away from the features.

An initial position is verified by checking for possible intersections between the two profiles involved. The computational expense for this is substantially reduced by first applying a Cyrus-Beck clipping algorithm to eliminate those void edges falling outside an approximate boundary enclosing the part [HILL90]. The convex clipping window required for this technique is provided in most cases by the simplified profiles used during part feature extraction. Otherwise a part's MER is used. All void edges not eliminated must be checked for possible intersections with the part using conventional methods.

All valid initial placements are further refined by shifting the part as far as possible in an *allowable shift direction* derived from the part orientation and the angles of the void feature involved. Figure 4.6 shows the three possible cases. For side one alignment, a shift direction toward the remaining resource is constructed by bisecting θ_{1V}. Bisection of θ_{2V} and θ_{BV} are used for alignment of the third and center sides, respectively. The part is translated in this direction until it contacts the void boundary. The contact vertices needed to initiate the INFP may then be found. The part is moved along the void boundary in either a clockwise or counter clockwise manner dependent upon which is consistent with the desired shift direction.

The INFP proceeds in a fashion similar to that described in Chapter Two; however, its construction over the entire remaining

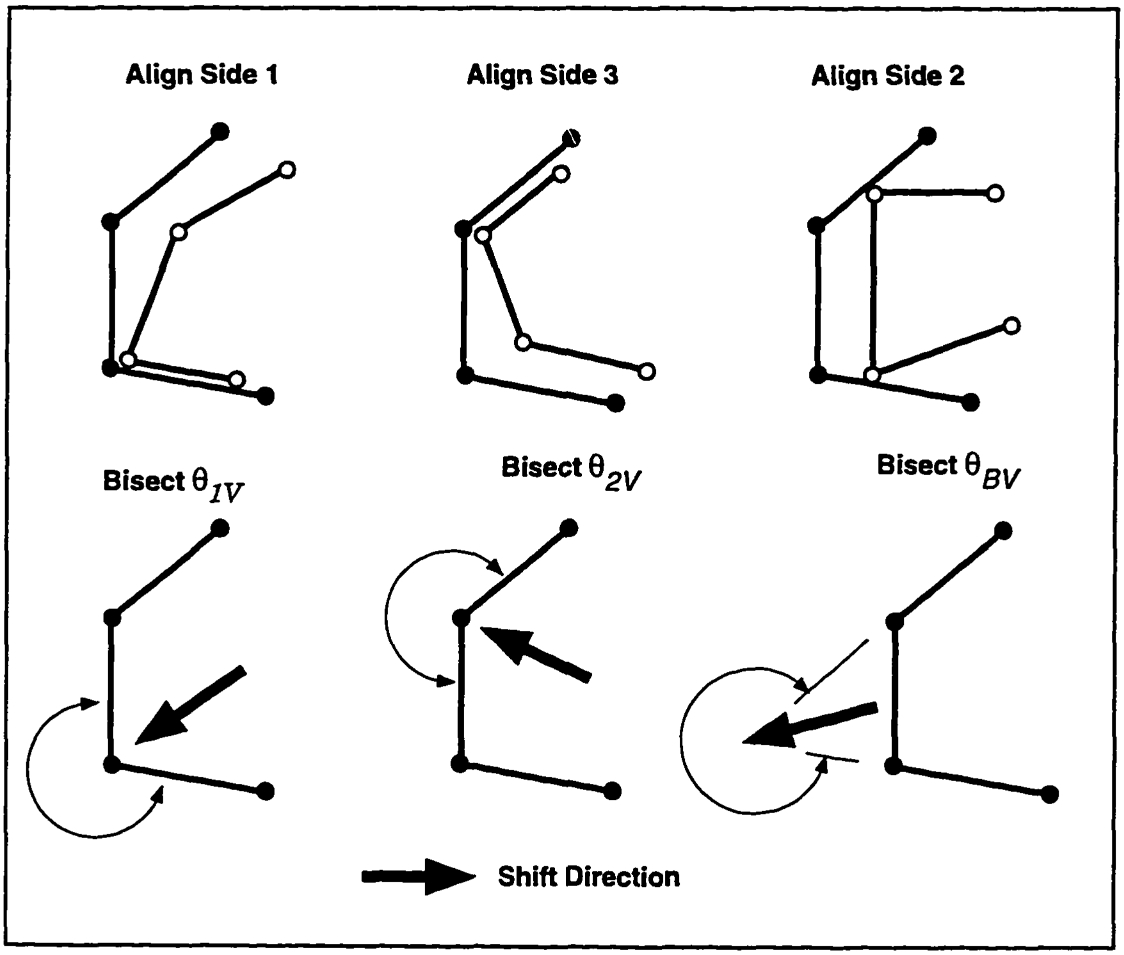


Figure 4.6 The three possible shift directions determined by the void feature angles and the aligned sides of the match.

resource boundary is not required. As before, movement vectors (§2.2.1) are provided by the edges of the void and part profiles. Shifting along the remaining resource boundary is permitted until a translation which opposes the allowable shift direction is detected by

$$\boldsymbol{S} \cdot \boldsymbol{S}_D < 0, \qquad [4.12]$$

being true, where $\boldsymbol{S}$ is a movement vector and $\boldsymbol{S}_D$ is the allowable shift direction described above. When this occurs, construction of the INFP is halted and the current part's position set as its *final placement*.

Two additional restrictions are also imposed during the part placement procedures. The first pertains to the amount of movement occurring between the initial to final placements. During INFP shifting, it is possible for the part to migrate beyond the region of the resource intended (Figure 4.7). In these cases, the benefit of matching the complementary shapes of the void and part features is often lost. These instances are detected and eliminated by limiting the total shift distance permitted to a pre-set percentage of the length of the part's MER.

The second restriction pertains to the allowable shift direction used in placement. A natural affinity exists for part placements along the long straight edges and square corners of the square resource stock due to the complementary shape reasoning heuristics implemented. Experience shows that this bias causes layout generation to proceed unfavorablely inward from the outer boundaries of the plate. To overcome this tendency, restraints are placed on the use of those *border features* containing elements coincident with the boundary of the original stock material.

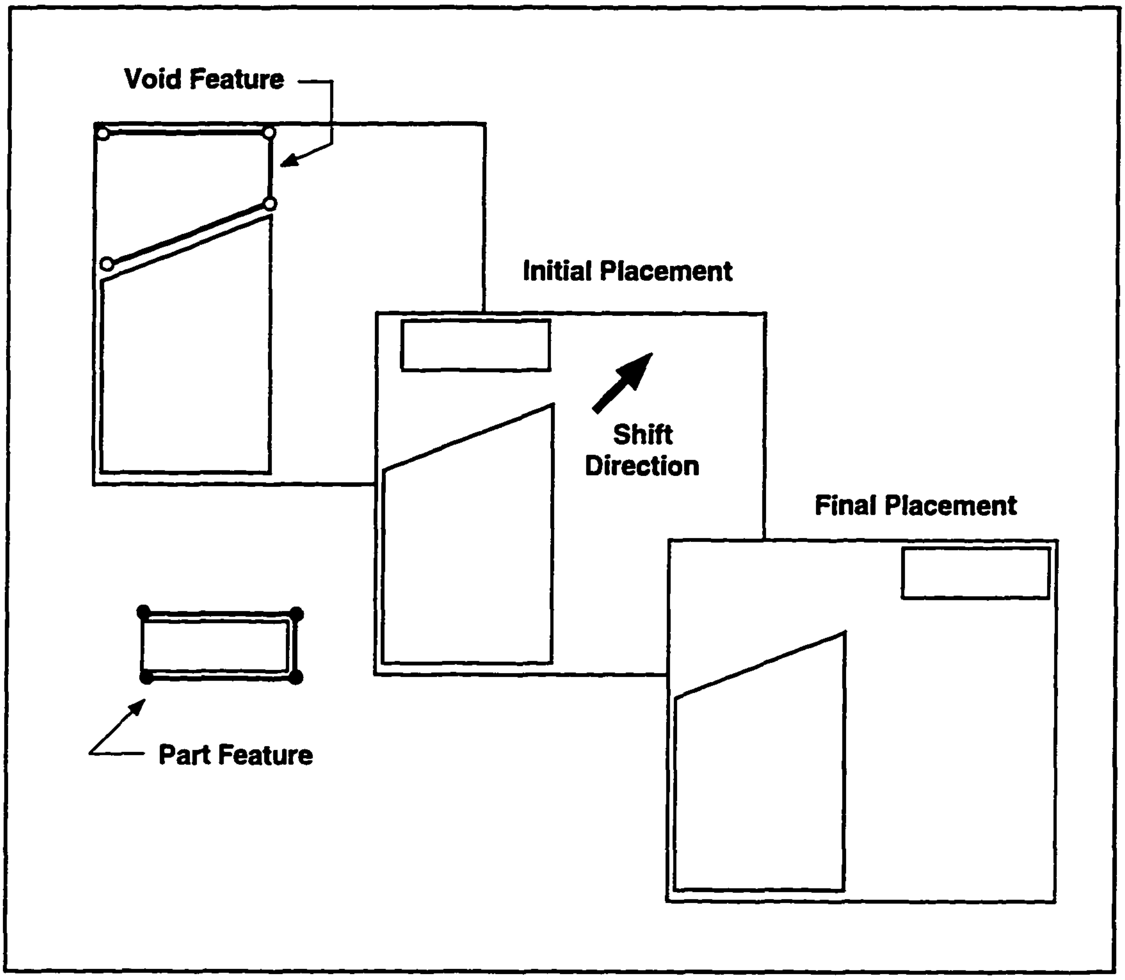


Figure 4.7 An example of the placement procedure where a part shifts beyond the intended region characterized by the void feature.

When such a border feature is involved, the allowable shift direction must fall between two limits, otherwise the match is ignored. These limits are referred to as the *border feature shift directions* and are defined by two angles measured from the horizontal, θ_{S_MIN} and θ_{S_MAX}. For example, parts will favor the upper left hand corner of a rectangular resource, when border feature shift directions are set between 90° and 180°. In such an arrangement, placements proceed from the left downward in a more beneficial manner. Additional control is also obtained by varying the acceptable shift direction range between part placements. The characteristics of the solution can be changed considerably using the border feature shift directions as will be shown in the next chapter.

To summarize, four steps are involved in placing a part (Figure 4.8).

1) Choose a primary side to orient the part with respect to the resource.
2) Determine the the align type for the selected features.
3) Locate the part using the align type and determine if it falls totally within the remaining resource.
4) Shift the part as far as possible within the void in the direction specified by the aligned sides.

4.3 Part Selection

As indicated above, given a matched pair of features it is possible to determine if a valid part placement or match exists. In practice, it is not unusual for each part to have five to ten features while the remaining resource may have more than twenty. The high volume of combinations makes it impractical and expensive to examine all possible part and void feature pairings. As a consequence a quick and computationally efficient method for determining the quality of the match between two features is needed. The strategy used is to calculate a single number measure or *matching index* for each possible alignment of a pair. The numerous matches can then be prioritized, and evaluations limited to those selected as best.

The matching index for a pair is evaluated using three basic geometric *difference measures* between the part and void features involved. These measures are calculated separately using the information stored with each feature, and are then combined to produce the single matching index. Since complementary shapes are sought, smaller differences are considered favorable. Consequently, the index is formulated such that lower values indicate a better match.

The first difference calculated is that between the *primary angles* of the match. The primary angle of each feature is included between its primary and secondary sides. The measure is calculated as:

$$\beta = \left| \theta_{(Primary\ V)} - \theta_{(Primary\ P)} \right| \ . \qquad [4.13]$$

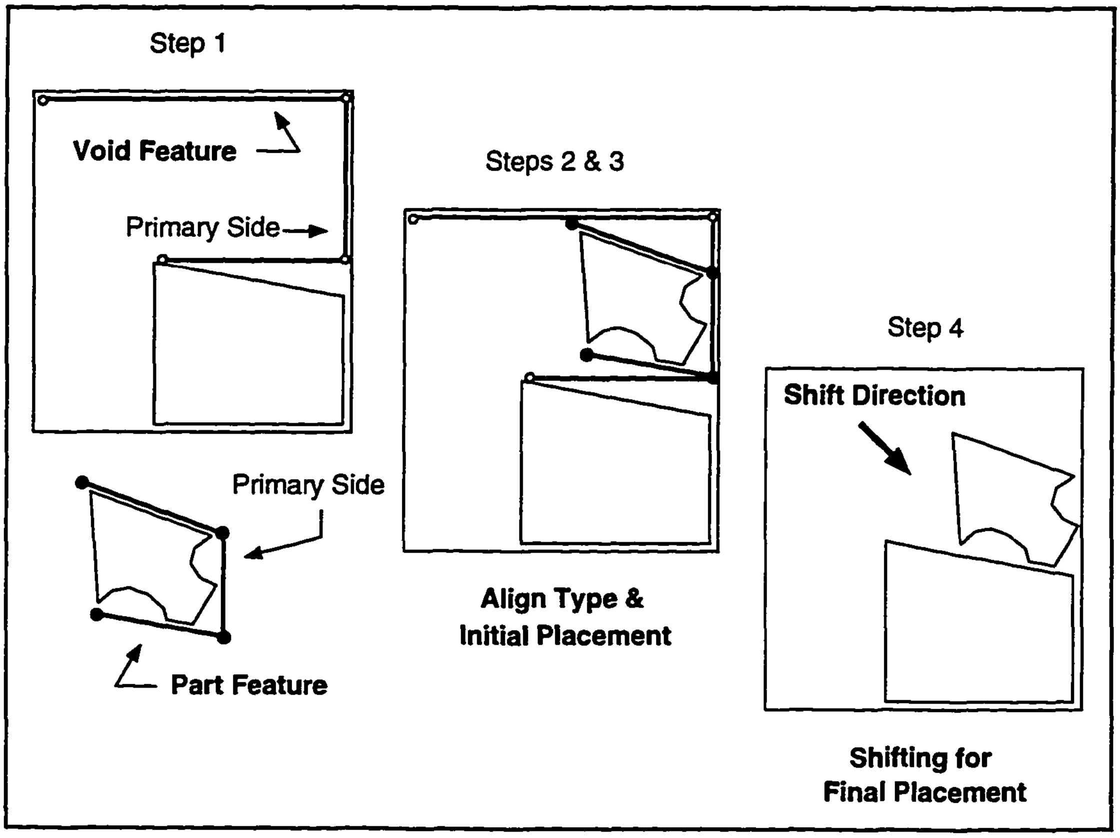


Figure 4.8 The four steps of part placement: 1) Select features and orient part, 2) Determine align type, 3) Initial placement, 4) Part shifting.

This difference is considered the most significant indicator of a good match since it corresponds to the angle between the void and part secondary sides (Figure 4.9). The value of β must fall below a tolerance (β_{TOL}), otherwise no index is produced and the match discarded. The role of matching index tolerances is discussed more fully in Chapter Five; however, the general intent is to eliminate the unnecessary calculation of unfavorable indices.

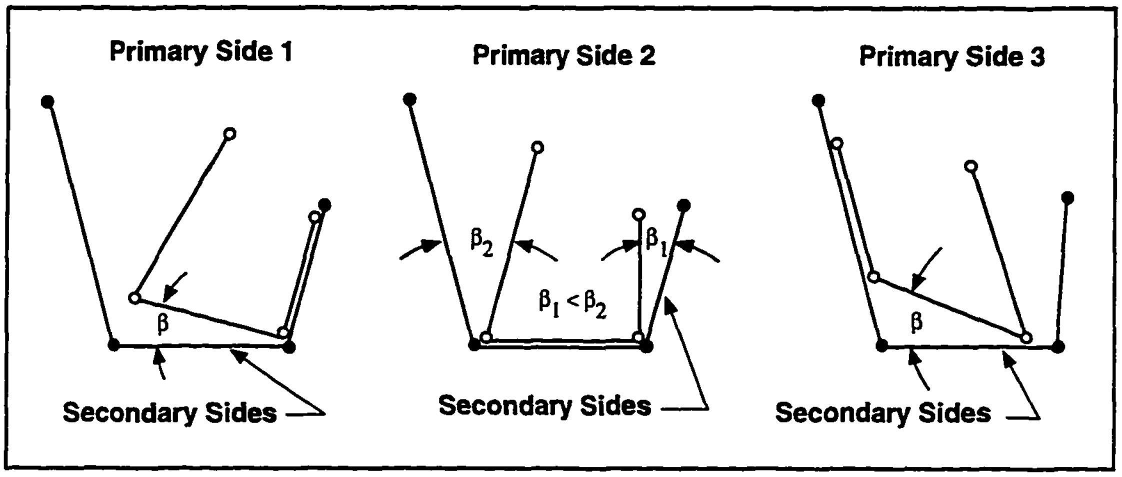


Figure 4.9 Angle β, the difference between the primary angles of a match for the three primary side alignments

If the calculated value of β is acceptable, the align type of the feature pair is determined. Only touching class align types are currently accepted, based on the premise that these types are the most likely to produce valid initial part placements. The arrangement of individual feature elements within the align types of this class also allow the straightforward calculation of meaningful difference measures. Similarities across the class can be exploited in the formulation of the matching index, permitting more consistent comparisons between the different align types. If the align type is accepted, its positioning information is stored and used for the calculation of the next difference measure.

The dissimilarity between the primary sides of a match is the second difference measure considered, and is represented as

$$X = ((l_{(Primary\ V)} - (l_{(Primary\ P)} + s_{AT})) \ / \ l_{(Primary\ V)} \ , \qquad [4.14]$$

where s_{AT} is the align type reference distance. For touching class align types s_{AT} indicates the relative position of the part primary side along the primary side of the void (Figure 4.10). A negative value of X implies extension beyond the primary side of the void, while a positive value indicates a short fall from its end. Both cases are expressed as a percentage measure through division by $l_{(Primary\ V)}$. The primary side difference is bounded by

$$X_{(TOL\ -)} < X < X_{(TOL+)} \qquad [4.15]$$

where each tolerance is set independently. Since extension beyond the void is more likely to cause an invalid part placement, the lower bound ($X_{(TOL\ -)}$) is typically made tighter. As before, any match falling outside the tolerances is discarded.

The final measure of difference examines the secondary sides of the match. Since they are not aligned, the projection of the part edge length onto the void indicated by the angle β is used for comparison (Figure 4.9). The difference is calculated as

$$Y = (l_{(Secondary\ V)} - l_{(Secondary\ P)} \cdot \cos\beta) \ / \ l_{(Secondary\ V)} \ . \qquad [4.16]$$

Because interaction of the secondary part side with the other feature entities is less conclusively known, no limits are set on this value. Since Y is the least significant of the difference measures, it is treated as a "tie breaker", and the matching index formulated to penalize exceptionally bad values.

The three raw difference measures are incorporated to form a single matching index. Recalling that a smaller index should indicate a better match, the measures are combined as follows.

$$MATCH\ INDEX = \frac{\beta}{\beta_{TOL}} + \left(\frac{X}{X_{TOL+}}\right)^2 + 0.2\left(\frac{Y}{Y_{TOL}}\right)^2 \qquad [4.17]$$

Within the formula, β and X are each normalized using their respective maximum limits. Since X is considered a less significant measure, its influence on the matching index is reduced by squaring its normalized value. The secondary side measure is also adjusted through division with Y_{TOL}, a desired limit set for that variable. Any Y values falling outside of the tolerance are automatically penalized when its adjusted value is squared ($Y/Y_{TOL} > 1$). Otherwise squaring lessens the effect of this term, as with X. The final change to Y's significance is made through a weighting factor of 0.2. In this way each difference measure is assigned its proper importance within the matching index.

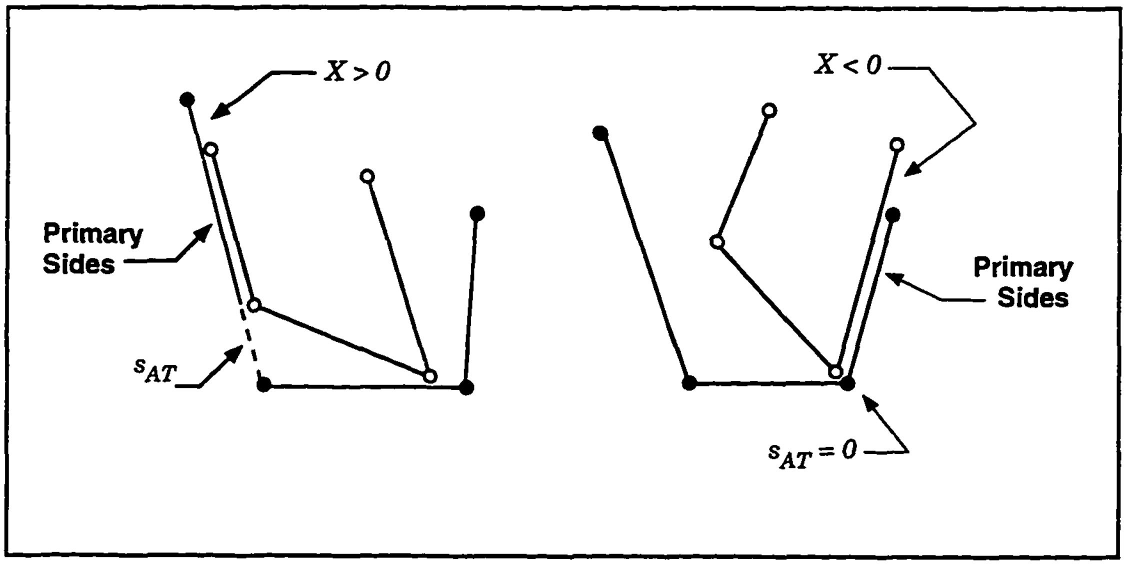


Figure 4.10 The positive and negative cases of the primary side measure of difference.

Through the use of matching indices, the relative value of all possible feature combinations can be quickly established. Part placement techniques can then be exercised to validate any selected match. The two basic tools for creating a layouts are now available. In the next chapter, these techniques are integrated with those for feature generation, to produce a comprehensive solution methodology to the bin packing problem.

Chapter 5
Implementation and Results

5.1 Introduction

In the first half of this chapter the overall solution strategy for nesting a complete set of parts is presented. The fundamental steps of the layout generation algorithm are outlined, demonstrating the integration of the feature generation, part placement, and part selection techniques previously detailed. The techniques are then combined to produce a comprehensive search algorithm which generates solutions through the iterative placement of parts onto a resource. Procedures for dynamically setting matching index tolerances, and the formulation of a waste function are then discussed. These two additional heuristics are required for the practical implementation of the solution technique.

In the remainder of the chapter results for a set of industry supplied part profiles are presented. A group of problems with diverse characteristics was selected to demonstrate the nature of the solution for various inputs. The resulting layouts are used to illustrate the robustness of the technique. By varying values, the influence of two key input parameters was also investigated. A series of layouts was generated and each parameter's effect upon the performance of the algorithm studied. Finally, the overall results for the feature based method were compared with those of the INFP technique used in the pilot study.

5.2 Search Algorithm

Figure 5.1 shows a flow diagram of the primary steps involved in generating solution layouts. Part profiles are first

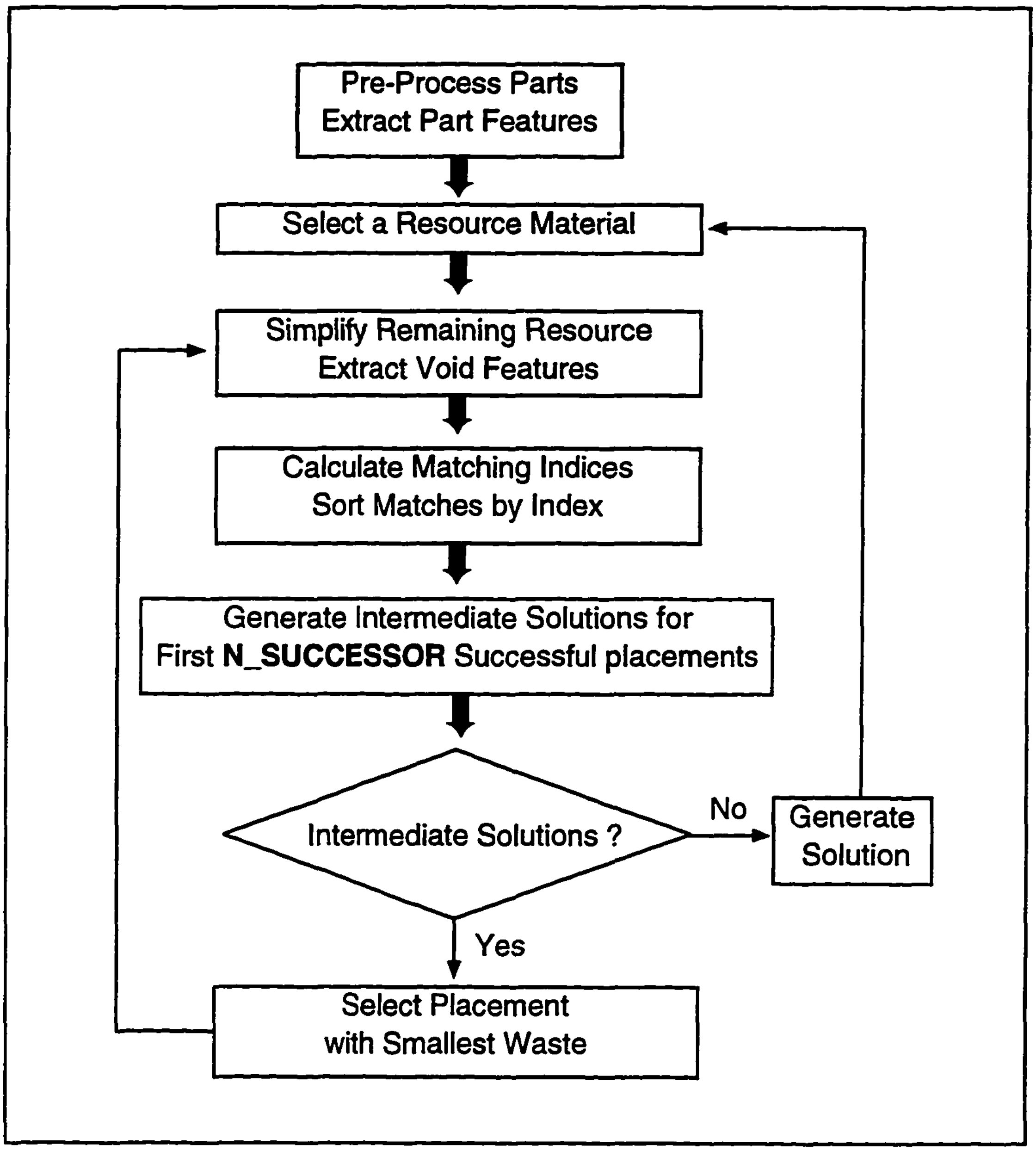


Figure 5.1 Flow diagram of the primary steps in feature based nesting

pre-processed. At this stage, curves are replaced with piecewise linear approximations, and if necessary for manufacturing, profiles are offset to maintain a specified distance between parts on the final layout. The primary role of preprocessing however, is to extract the features representing the parts at varying levels of detail. This extraction of part features is performed only once as their properties are invariant over the course of the solution. The geometric information associated with each feature is stored in a data base which is accessed repeatedly during processing.

Following preprocessing, the stock material for the layout is selected. The algorithm then advances into the main decision loop where the majority of work is preformed. Each iteration of this loop represents the addition of one part to the final solution.

The first step in placing the next part is the simplification of the resource remaining from the previous stage. Initially, this resource is the entire stock material. To achieve the appropriate degree of simplification, a new set of LENGTH_TOL and AREA_TOL values are first calculated based on those parts still eligible for placement. Recalling that three tolerance pairs representing the smallest, middle and largest parts are used (§3.3.2), the SCR and LSRD techniques are applied to produce simplifications of the remaining resource at varying levels of detail. The current void features are then extracted and stored.

Matching indices for all possible feature combinations are now generated from the part and void data base. Progressing from most to least favorable indices, the placements indicated by each pairwise match are examined and invalid placements due to overlap eliminated. Each valid placement is called a *successor* of the current solution, and its associated part orientation and location are saved. Matches are tested until a preset number of successors (*N_SUCCESSOR*) are generated for the intermediate solution. However, only one is used for further expansion. Selection of this most favorable placement is based on the waste function described in the next section.

Solution layouts are constructed through the repeated application of the three primary steps: void feature extraction, matching index calculation, and part placement generation. Each intermediate stage is initiated using the remaining resource of the most recently chosen successor. If no successors exist, additional parts cannot be placed on the resource. The current solution is then output and nesting started on a new stock blank. The complete solution is realized when all parts have been allocated.

5.3 Waste Function

At each intermediate stage of the solution a single successor must be chosen for further expansion. The relative value of each part placement is distinguished by calculating a waste function for each of the successors generated. Allocations with smaller waste values are considered more favorable. The appraisal of quality is based on two terms assessing the current waste and predicting the future waste of a solution. *Future waste* is an estimate of the penalty incurred by allocating all remaining parts, while *current waste* deals with the actual trim loss on the existing layout. This distinction is similar to that made by Albano and Sappupo, however, more intelligent heuristics are used to calculate each term [ALBA80].

Future waste is calculated as a set percentage of the concave and true (i.e. actual) areas of all unplaced parts. Each part's *concave area* is defined as the region between its exact and convex hull profile. For the current application the *individual future waste* of a part is calculated as

$$(\mathit{FUTURE\ WASTE})_i = 1.0 \cdot (\mathit{True\ Area})_i + 0.4 \cdot (\mathit{Concave\ Area})_i . \quad [5.1]$$

Total future waste for an intermediate solution is taken as the summation of individual wastes over the set of unplaced parts. As a part's future waste is eliminated once it has been allocated, this approach favors the early and consequently easier placement of larger and more complex profiles. This is desirable as experience shows such parts are generally more difficult to place.

Current waste consists of two terms, true waste and unusable area. *True waste* is the areas or gaps trapped between parts, which are no longer contained in the remaining resource and are not accessible for nesting (Figure 5.2). *Unusable areas* are the small recesses, notches, chamfers and necks eliminated during a void profile simplification. To detect these regions, the SCR and LSRD simplification procedures are carried out on the remaining resource of each successor. The controlling

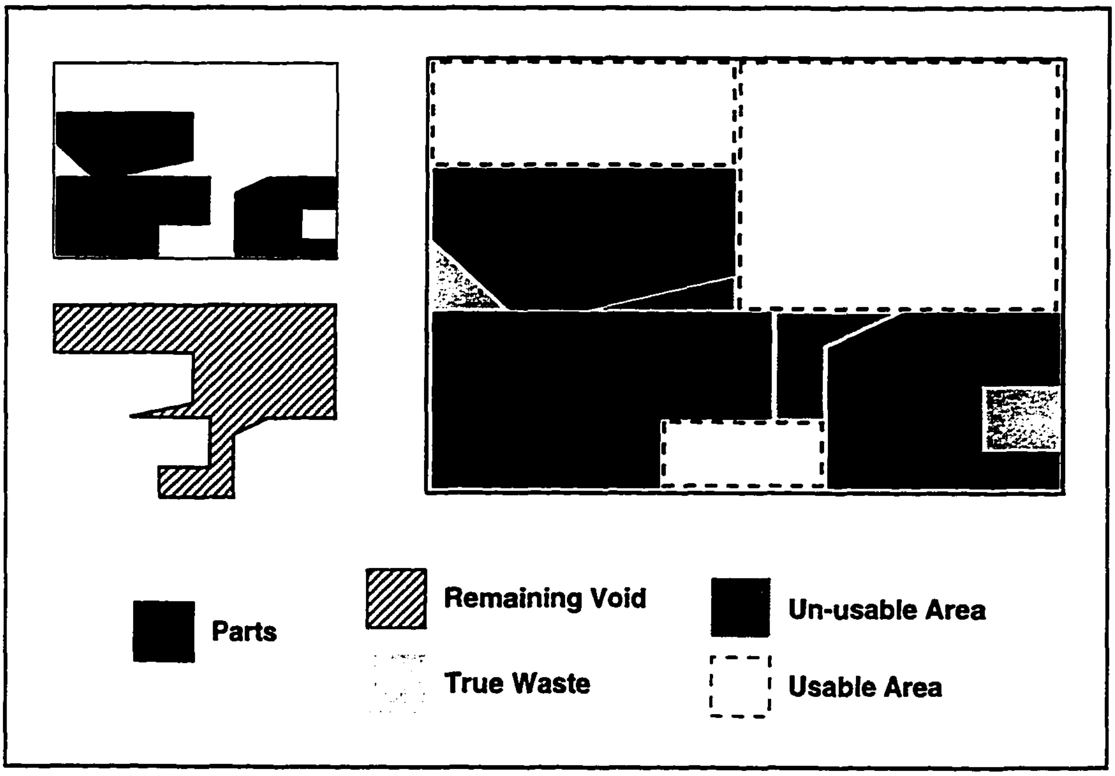


Figure 5.2 Elements of current waste

LENGTH_TOL and AREA_TOL parameter pair required is calculated using only the smallest third of remaining parts. This policy prevents the unnecessary inclusion of waste from areas possibly usable by smaller parts. Although technically contained within the remaining resource, unusable area is considered inaccessible based on the size and shape of the unplaced parts. The regions remaining after simplification are referred to as *usable areas*.

The distinction between good and bad placements is often unclear when based solely upon future waste, true waste, and unusable area. For example, Figure 5.3 demonstrates two placements of varying quality which generate equal waste measures. For this reason a *perimeter penalty* is calculated to provide additional information for each successor. Although this term is used to scale the unusable area, it is

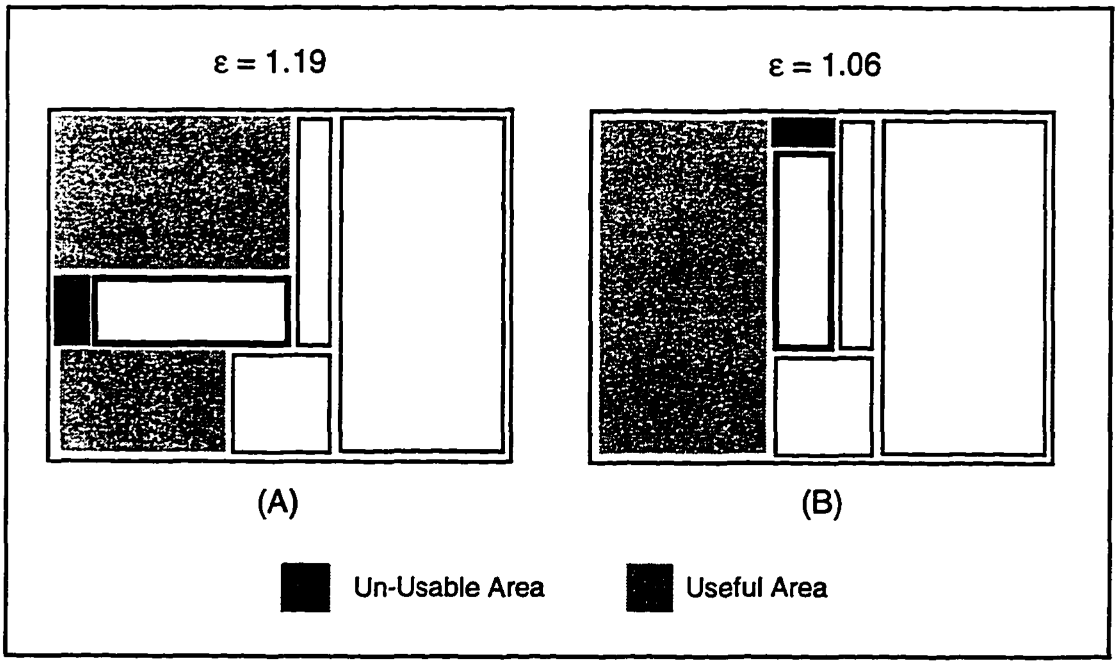


Figure 5.3 Two placements of the same part generating equal amounts of un-usable and useful area. Case A has the higher perimeter penalty.

calculated by examining the size and shape of usable subregions produced duing remaining resource simplification. The formulation is:

$$\varepsilon = \frac{2 \cdot \Sigma(A_i)}{\sum\left(\frac{\Phi(A_i)}{P_i} A_i\right) + \left(\frac{\Phi(\Sigma A_i)}{\Sigma(P_i)}\right) \Sigma A_i} \qquad [5.2]$$

where

P_i	Perimeter of the i_{th} Usable Area,
A_i	Area of the i_{th} Usable Area,
$\Phi(x)$	Returns Perimeter of Square of Area x.

All summations are taken over the set of usable subregions.

The two ratios in the denominator are the key elements of Equation 5.2. The first ratio, $\Phi(A_i)/P_i$, is used to measure the "square-ness" of each *individual* subregion by comparing its perimeter to that of a square of equal area. Each of these ratios is then weighted based on the corresponding subregion's area. By contrast, the second ratio, $\Phi(\Sigma A_i)/\Sigma(P_i)$, examines the summation of *all* subregion perimeters. This sum is compared to the perimeter of a single square equaling the area of the combined usable regions. The remaining terms in this metric are used to scale the penalty such that a value of unity is achieved for the optimal case. The overall effect of this formulation is to favor a single rectangular region over several small irregular ones. Incorporating all terms, the total waste is calculated as:

$$\begin{aligned} \mathit{TOTAL\ WASTE} = {} & \mathit{FUTURE\ WASTE} \\ & + 1.5 \cdot (\mathit{TRUE\ WASTE}) \\ & + \varepsilon \cdot (\mathit{UN\text{-}USABLE\ AREA}) . \end{aligned} \qquad [5.3]$$

5.4 Matching Index Tolerances

During the solution process, the total number of potential matches between part and void features can be very large. For sizable problems, sets of five hundred or more part features and twenty or more void features are common. Also recall that for each pair of void and part features, there are three matches corresponding to the three allowed edge orientations. Such a situation would require the calculation and storage of thirty thousand matching indices. Since the majority of these represent invalid placements and only a limited number of successors are produced at each stage of the solution, substantially fewer matching indices need to be calculated. This is accomplished through matching index tolerancing.

A large portion of potential combinations are eliminated by limiting the values of the difference measures β and X used to calculate the matching index. As described earlier (§4.3) these value are restricted as

$$\beta < \beta_{(TOL)i} \qquad [5.4]$$

and

$$X_{(TOL-)i} < X < X_{(TOL+)i} \, , \qquad [5.5]$$

where the subscript i refers to a tolerance level dynamically set throughout the solution. Table 5.1 shows the range of values available for the current application including those for the parameter Y_{TOL} (Equation 4.17). For the initial allocation of each resource, tolerances are set to their most restrictive values, as the pool of available part features is generally large. After each stage of the solution, the number of successors generated is examined. If the desired number was produced, the tolerances are tightened while remaining in the range of tabulated values. Otherwise they are relaxed to provide a larger pool of potential matches.

If no valid placements can be produced from the set of generated indices, all restrictions on matches are relaxed, and a new set of matching indices calculated and investigated. This includes all matching index tolerance values and any potential restriction imposed by the border feature shift directions (§4.2.3). This insures that all possible placements are tested and sometimes permits additional allocations.

Table 5.1 Matching Index Tolerance Values

	Tolerance Level i	$\beta_{(TOL)i}$	$X_{(TOL+)i}$	$X_{(TOL-)i}$	$Y_{(TOL)i}$
More Restrictive	1	0.1	0.80	-0.10	0.20
↓	2	0.2	0.90	-0.15	0.40
Less Restrictive	3	0.4	0.95	-0.20	0.60

5.5 Results

The shape reasoning methodology introduced has been implemented and tested extensively. All algorithms were coded in the C programming language, and the application run on a VAX 4000-300 server under the VMS operating system. With the exception of input and output file specification, the package is totally automated.

An industrial marine fabricator provided a diverse set of 42 actual problems to evaluate the robustness of the approach. Each problem included a data file defining the geometry of all unique or *master* parts and a list of their instances. Part descriptions consisted of straight lines and circular segments which were approximated with the appropriate inscribed (voids) or circumscribed (parts) chords. To reduce the computational expense of all profile operations, each part was simplified using the modified convex hull approach of the pilot study [LAMO96] and also offset a prescribed amount to account for flame-cutting fabrication. Although original part boundaries are plotted in the solution layouts, only these more practical approximate offset representations are used during processing.

During preprocessing, the internal voids of larger parts are extracted and ordered in decreasing size for possible use as irregular resources. These regions are nested first. All profiles remaining are then allocated to stock standard rectangular plates selected successively from a prioritized list of available materials provided for each problem by industry. The voids and any associated nests are then re-inserted onto their parent parts when generating output files.

To help quantify the characteristics of the individual problems investigated, a series of criteria were developed. In addition to total part count and number of masters, four measures describing the profiles contained in the problem were defined. Illustrated graphically in Figure 5.4, these are:

$$\text{Normalized Area} = \frac{\text{Area of Part Profile}}{\text{Area of Stock Resource}}$$

$$\text{Irregularity} = \frac{\text{Area of Part MER}}{\text{Area of Part Profile}}$$

$$\text{Concavity} = \frac{\text{Area of Part Convex Hull}}{\text{Area of Part Profile}}$$

Complexity = Number of vertices in profile description

Normalized area gauges the size of a part relative to the selected resource. Irregularity measures the non-rectangularity of a profile [BRON83], while concavity indicates the degree of divergence from its convex hull. Complexity refers to the number of vertices in the simplified profile used for processing.

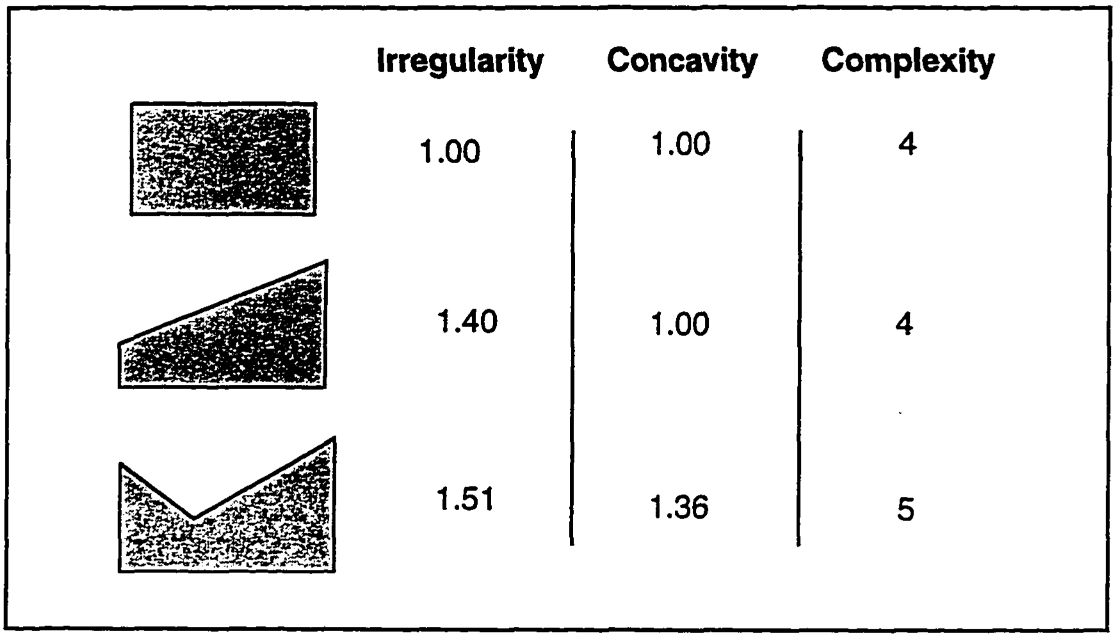


Figure 5.4 Examples of the measures used to describe each part profile

For brevity, mean values of these measures for an entire profile set are usually presented. However, averages are sometimes not representative of the problem nature, particularly when distributions across the part set are non-uniform. If prompted, the application will furnish a more detailed problem description, including histograms to demonstrate the precise distribution of profile characteristics. Additional information pertaining to aspect ratios and part duplications are also provided. A problem description of this type is displayed in Appendix A.2.

5.5.1 Parameter Study

The integral parameters and tolerances of the various techniques described throughout this document are the sole method for controlling the algorithm performance. When possible, these values are calculated by the application, and are allowed to change dynamically during processing, while others are static and input at run time. In some respects, the method is very sensitive to these inputs. A change in parameter values may cause the placement of an initial part on a plate to alter only slightly. This relatively minor change will cascade throughout all of the remaining steps, making solutions for the same problem set often appear totally unrelated. However, when examining results for an entire set of problems, small changes do not substantially affect overall waste and CPU times.

The parameter values currently used are those selected from experience to produce good overall results for a broad range of problems. Reasonable ranges for most parameters can be determined in a relatively short time through comparative runs of the program.

Two key parameters were selected to demonstrate the nature of the solution and the influence of input on the performance of the algorithm. Two values for the number of successors and the range of border feature shift directions were chosen. Several runs of the application were conducted to investigate the four possible permutations. The two cases for shift direction correspond to a constant range of 180° to 360° and a case alternating between the ranges of 90° to 180° and 180° to 270°. The number of successors was set to 10 and 20. Experience has shown these values produce reasonable results for rectangular resources.

To reduce the CPU time and analysis overhead, a meaningful and manageable subset of the original 42 industry problems was used to conduct the parameter study. A heterogeneous group of profile sets was selected to portray the full diversity associated with the bin packing class of problem. Table 5.2 shows the characteristics of these problems. Tables 5.3 and 5.4 summarize the results from the four cases, including total CPU time and the number of stock plates used.

For the majority of problems, cumulative waste is identical in each of the four cases, since nesting the total part set requires the same number of stock plates. A better indication of quality is obtained by examining the percent usage for the last plate. A lower value here indicates fewer profiles on the final plate, implying better initial packing. Also, more material remains for nesting of additional parts should they

Table 5.2 Problem set overall characteristics

Prob. Set	Number of Parts	Number of Masters	Average Normalized Area x 100	Total Normalized Area x 100	Average Irregularity	Average Concavity	Average Complexity	Stock Height (cm)	Stock Width (cm)
A	70	50	7.55	528	1.33	1.00	12.2	307	1280
B	109	70	19.48	2124	1.15	1.00	11.0	307	1402
C	206	70	1.84	378	1.18	1.00	6.6	183	914
D	103	60	3.57	368	1.40	1.11	8.3	305	1219
E	295	197	2.41	711	1.05	1.03	6.0	305	1219
F	52	31	5.05	263	1.21	1.12	6.6	305	1219
G	84	49	2.95	247	1.00	1.00	5.5	305	1219
H	56	35	6.37	357	1.39	1.18	12.1	305	1219
I	107	57	6.65	711	1.25	1.08	8.3	305	1219
J	111	50	5.21	578	1.18	1.10	8.2	305	1219
K	37	21	2.85	106	1.08	1.01	5.7	305	1219
L	52	35	7.42	386	1.17	1.05	7.6	305	1219
M	68	32	2.60	177	1.13	1.07	6.6	305	1219

exist. The feature based method is also formulated to conveniently nest on large irregular sections of trim waste, should they be retained for later use.

The parameter study problems were also nested using the pilot study application. Results are reported in the tables indicated above, for comparison to the feature based solutions. In all cases, backtracking within the search algorithm was disabled by setting the bandwidth to zero (§2.2.2). Analysis from the pilot study indicated that backtracking substantially increased computing time and did not consistently reduce trim waste. This conclusion agreed with that reported by Albano

Table 5.3 Summary of results with border restrictions set to 180°→360°. Asterisks denote feature based solutions requiring fewer plates than the NFP technique.

Problem Set	Total CPU time (sec)			Total Plates Used			% Usage Final Plate		
	N_SUCCESSOR		NFP	N_SUCCESSOR		NFP	N_SUCCESSOR		NFP
	10	20		10	20		10	20	
A	967	1283	1698	9	9	9	40.5	40.1	41.0
B	451	723	993	30	29	30	18.4	65.8*	26.2
C	2665	5406	5899	5	5	5	41.3	36.2	45.7
D	1773	2235	3579	5	5	5	54.4	53.6	50.2
E	2912	5852	8889	9	8	9	1.3	53.6*	4.4
F	240	380	239	4	4	4	13.3	6.4	20.0
G	285	388	320	3	3	3	53.8	46.2	55.7
H	407	821	1296	6	5	6	10.7	68.1*	11.7
I	664	1128	1629	10	10	10	9.0	4.0	38.0
J	1058	1999	3691	7	8	8	78.7*	12.9	17.7
K	73	117	147	2	2	2	13.5	12.4	21.0
L	266	411	394	5	5	5	60.2	60.4	64.9
M	192	397	586	3	3	3	4.6	5.9	11.6

and Sapuppo [ALBA80]. Elimination of the backtracking component also permitted a more straightforward comparison of the two methods, based on the merits of the NFP and shape based techniques.

Several observations can be made from Table 5.5, which shows normalized CPU times for all cases of the study. Computation times for the feature based solutions are lower than those of the NFP technique. When using an NFP approach, a tentative placement for *each* of the remaining available parts must be generated at every stage of the solution. In theory, CPU time increases quadratically as a function of the

Table 5.4 Summary of results with border restrictions alternating between 90°→180° and 180°→270°. Asterisks denote feature based solutions requiring fewer plates than the NFP technique.

Problem Set	Total CPU time (sec)			Total Plates Used			% Usage Final Plate		
	N_SUCCESSOR		NFP	N_SUCCESSOR		NFP	N_SUCCESSOR		NFP
	10	20		10	20		10	20	
A	459	1000	1698	9	9	9	40.5	40.5	41.0
B	462	674	993	29	29	30	65.8*	65.8*	26.2
C	2062	3040	5899	5	5	5	31.2	34.1	45.7
D	977	1880	3579	5	5	5	54.6	37.1	50.2
E	1337	3339	8889	8	8	9	56.2*	45.8*	4.4
F	139	225	239	4	4	4	9.7	6.4	20.0
G	159	323	320	3	3	3	51.1	47.9	55.7
H	279	440	1296	6	5	6	2.7	61.9*	11.7
I	453	814	1629	10	9	10	10.7	62.3*	38.0
J	603	1999	3691	8	8	8	12.2	12.9	17.7
K	43	117	147	2	2	2	12.6	12.4	21.0
L	185	411	394	5	5	5	60.2	60.4	64.9
M	173	397	586	3	3	3	10.1	5.9	11.6

total number of profiles nested (§2.4). Over the course of a solution, a decrease in CPU time from plate to plate is evident, as a decreasing number of parts are available. This trend is seen in Tables 5.6 and 5.7 where CPU times per part are shown for each plate in the NFP solution for two representative problems. The corresponding layouts are depicted in Figures 5.5 and 5.6.

For the shape based heuristic an increase in parts corresponds to an increased number of matching indices. Calculating indices accounts for a small percentage (5% to 10%) of total CPU time. For the

Table 5.5 Summary of CPU time per part reslut for the parameter study

Problem Set	CPU Time Per Part (sec)					Number of Parts
	N_SUCCESSOR = 10		N_SUCCESSOR = 20		NFP	
	180° → 360°	90°→180° ⇔ 180°→270°	180° → 360°	90°→180° ⇔ 180°→270°		
A	13.81	6.56	18.33	14.29	24.26	70
B	4.14	4.24	6.63	6.18	9.11	109
C	12.94	10.01	26.24	14.76	28.64	206
D	17.21	9.49	21.70	18.25	34.77	103
E	9.87	4.53	19.84	11.32	30.13	295
F	4.62	2.67	7.31	4.33	4.60	52
G	3.39	1.90	4.62	3.85	3.81	84
H	7.27	4.98	14.66	7.86	23.14	56
I	6.21	4.23	10.54	7.61	15.22	107
J	9.53	5.43	18.01	18.01	33.25	111
K	1.97	1.16	3.16	3.16	3.97	37
L	5.12	3.56	7.90	7.90	7.58	52
M	2.82	2.54	5.84	5.84	8.62	68
TOTALS	8.85	5.43	15.66	10.86	21.75	1350

current application, evaluation of the successor waste functions represent the largest portion of computation time (60%-80%). However, since at each stage these calculations are capped by the total number of allowed successors (*N_SUCCESSOR*), solution times are less adversely effected by the number of parts. CPU times per part remain relatively equal over the course of a solution, while total CPU time roughly doubles as the number of successors varies from 10 to 20.

Table 5.6 Individual plate results for problem set C (Figure 5.5). N_SUCCESSOR equals 20.

Plate Number	CPU Time Per Part			Number of Parts			% Trim Loss		
	180° → 360°	90°→180° ⇔ 180°→270°	NFP	180° → 360°	90°→180° ⇔ 180°→270°	NFP	180° → 360°	90°→180° ⇔ 180°→270°	NFP
1	29.0	12.4	45.8	53	57	64	20.29	21.74	23.91
2	16.6	27.2	31.0	49	57	66	18.12	21.74	26.81
3	38.2	8.8	26.7	56	51	26	28.99	26.81	34.06
4	21.4	9.2	6.5	33	27	25	34.78	29.71	26.81
5	14.3	6.1	2.6	15	14	25	63.77	65.94	54.35

Table 5.7 Individual plate results for problem set H (Figure 5.6). N_SUCCESSOR equals 20.

Plate Number	CPU Time Per Part			Number of Parts			% Trim Loss		
	180° → 360°	90°→180° ⇔ 180°→270°	NFP	180° → 360°	90°→180° ⇔ 180°→270°	NFP	180° → 360°	90°→180° ⇔ 180°→270°	NFP
1	18.1	8.0	30.9	10	15	12	40.1	31.37	28.98
2	16.0	6.0	33.0	13	10	14	38.3	35.73	37.47
3	12.3	10.8	24.5	11	12	11	30.8	26.36	37.15
4	15.9	6.0	10.7	10	10	11	23.1	32.68	35.62
5	11.6	7.8	12.6	12	9	6	31.9	38.13	36.71
6	N/A	N/A	0.3	N/A	N/A	2	N/A	N/A	88.34

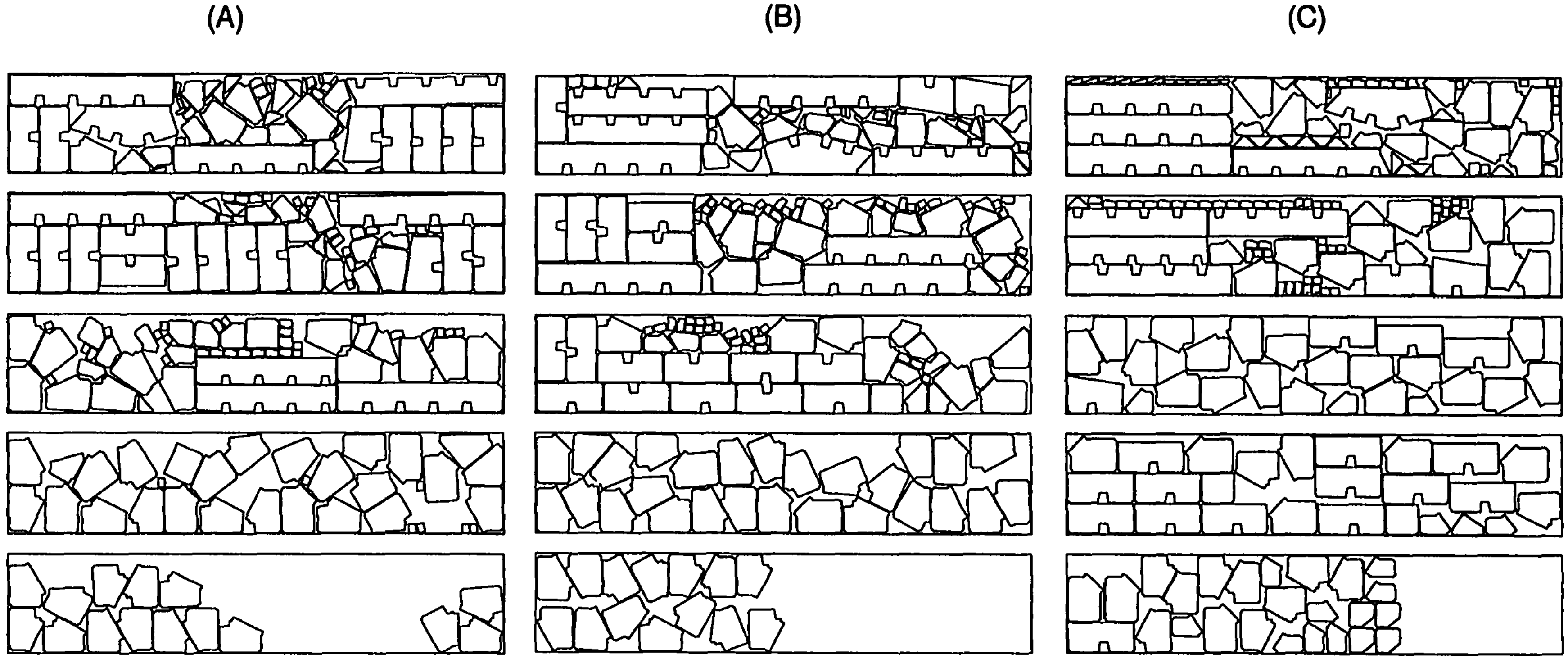


Figure 5.5 Solutions for Problem C. Case (A): N_SUCCESSOR = 20, Border Restrictions 180°→360°. Case (B): N_SUCCESSOR = 20, Border Restrictions alternate between 90°→180° & 180°→270°. Case (C): Pilot study results.

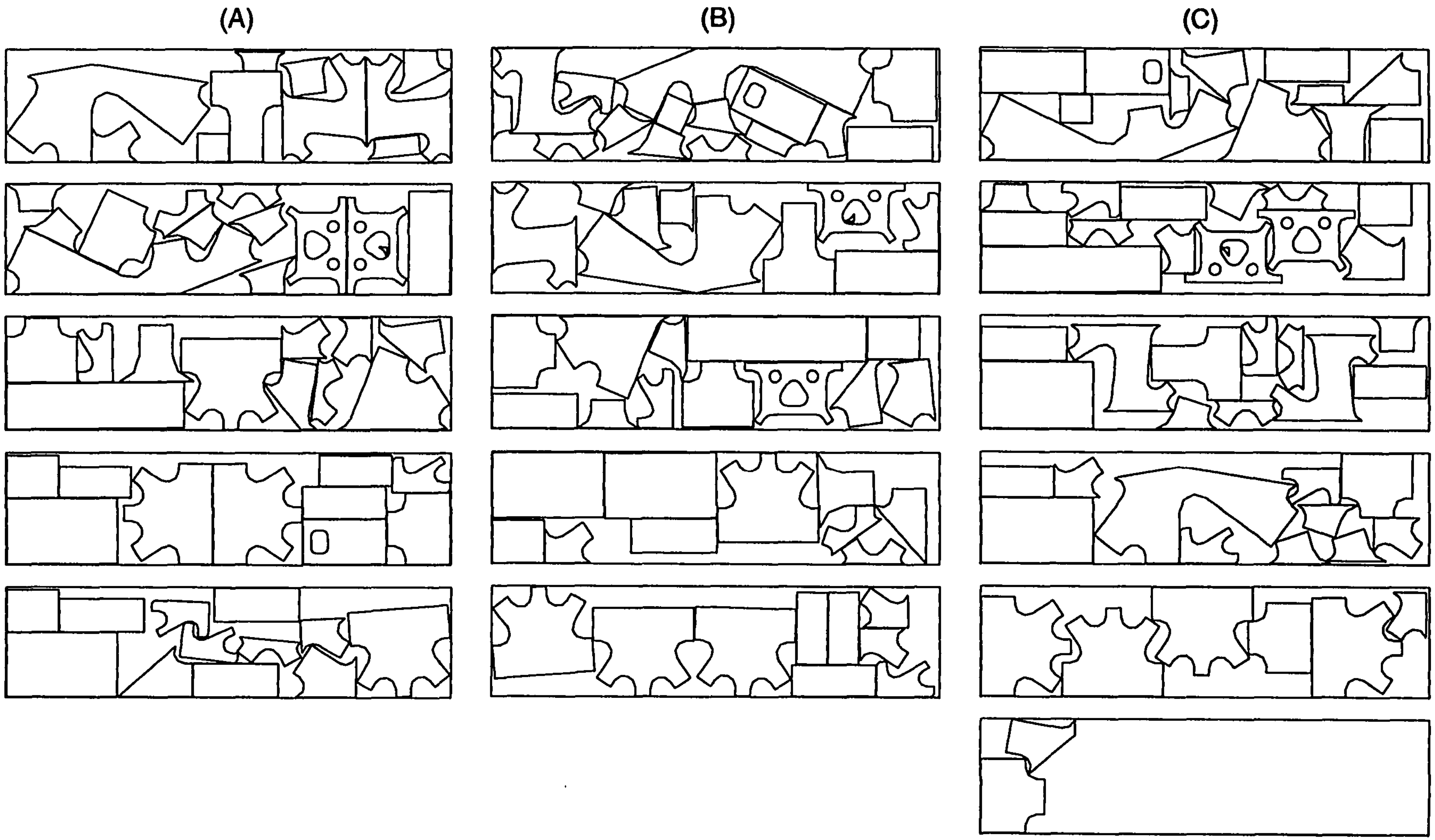


Figure 5.6 Solutions for Problem H. Case (A): N_SUCCESSOR = 20, Border Restrictions 180°→360°. Case (B): N_SUCCESSOR = 20, Border Restrictions alternate between 90°→180° & 180°→270°. Case (C): Pilot study results.

As seen in Tables 5.2 and 5.3, reducing the number of successors will sometimes adversely affect the solution. Placements are generated in order from most to least favorable matching index. However, since the indices are only an estimate of the quality of fit between the part and void, there is no guarantee that the initial placements indicated will be best. Consequently, an adequate pool of valid placements from which to choose must be generated. Extra successors raise the probability of finding a better solution, however the additional benefit diminishes with each new placement. In practice the value of N_SUCCESSOR is set to produce a balance between the required quality of nest and the desired computation time.

Interestingly, the increased computation time associated with more successors does not always produce improved layouts. An example is shown in Figure 5.7, where one less stock plate is needed by the solution accessing ten fewer successors at each stage of the solution. Such inconsistencies are related to the difficulty of predicting the future effect of each successor. The waste function must forecast each placement's influence on future allocations, based solely upon the shape of the current remaining resource. As a result, the generation of additional successors will occasionally allow the selection and expansion a less favorable solution.

Another trend seen in the parameter study data were smaller CPU times for cases where border shift restrictions alternated between 90°to 180° and 180° to 270°. This result can be explained in part by the effect of the restrictions on the formation of the remaining resource. A single range of 180° to 360° forces parts to migrate toward the

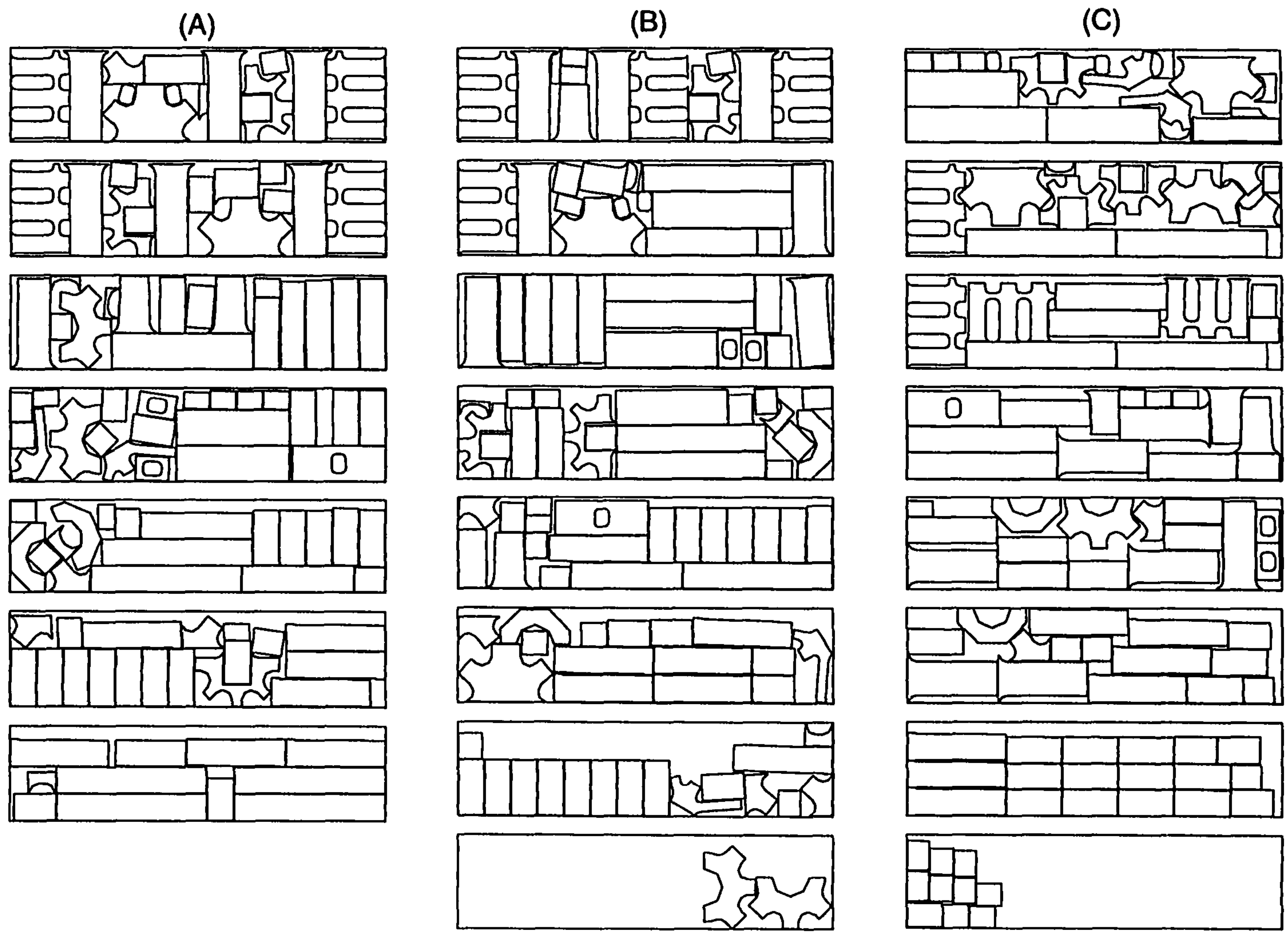


Figure 5.7 Solutions for Problem J. Border Restrictions are 180°→360°. Case (A): N_SUCCESSOR = 10. Case (B): N_SUCCESSOR = 20. Case (C): Pilot study results.

lower corners of the stock (Figure 5.5 case A). By contrast, the alternating case causes parts to favor the left side of the plate (Figure 5.5 case B). In the former situation, the leading edge of the solution tends to extend along the length (longer side) of the plate, while for the later it spans the height (shorter side). Remaining voids for the 180° to 360° case are generally more complex and require larger CPU times for evaluation of their waste functions.

5.5.2 Analysis of All Cases

Based on the result of the parameter study a complete run of the 42 industry problems was conducted with 20 successors and alternating shift restrictions as above. Number of parts, computation time, total resources required, and percent usage on the final plate are shown for each problem in Table 5.8, along with values from the pilot study application.

Total CPU time to place the 3,014 parts was 8.06 hours for the feature based method, while the NFP technique required 11.46. For 35 of the problems, both methods required the same number of stock plates. In six of the remaining seven problems, the feature based technique required one less resource. The NFP technique used fewer stock plates (1) in only one of the 42 problems. Representative examples of the feature based solutions are plotted in Appendix A.3, with corresponding examples of the NFP method found in A.4.

Material usage on final plates for both methods was also investigated. Data from both techniques was adjusted to produce a valid comparison. When either method required one less resource, percent usage on the final plate for that technique was considered zero. This is a conservative measure as it ignores the available material remaining on that solution's true last plate. Solutions requiring one plate were ignored, as material usage is identical for both methods and does not indicate the quality of packing achieved. Using these analysis criteria an average material savings of 4 percent was realized on the final plate of feature based solutions.

Table 5.8 Results for all cases. N_SUCCESSOR equals 20. Border restrictions alternate between 90°→180° and 180°→270°.

Problem Set	Number of Parts	Total CPU time (sec)		Total Plates Used		% Usage Final Plate	
		SHAPE Based	NFP Based	SHAPE Based	NFP Based	SHAPE Based	NFP Based
1-A	70	1300	992	9	9	40.5	41.0
2-B	109	694	993	29	30	65.8*	26.2
3	411	2355	751	56	55	26.9	26.9*
4	10	35	6	3	3	2.8	1.3
5	42	347	101	2	3	81.9*	9.4
6-C	206	3080	5899	5	5	34.1	45.7
7	6	50	1	1	1	8.0	8.0
8	15	65	8	2	2	42.8	39.1
9	156	1580	864	6	6	49.3	38.4
10	1	1	1	1	1	45.7	45.7
11	344	2602	4107	14	15	63.0*	11.4
12	4	7	1	4	4	44.0	44.0
13-D	103	1837	3577	5	5	37.1	50.2
14	34	376	269	3	3	8.7	8.8
15	3	14	1	1	1	9.5	9.5
16	76	887	778	2	2	60.8	63.6
17-E	295	4915	8889	8	9	45.8*	4.4
18	4	38	1	1	1	9.8	9.8
19	76	831	1226	5	5	61.2	55.0

* Required one less plate than other corresponding solution

(table con'd.)

Problem Set	Number of Parts	Total CPU time (sec)		Total Plates Used		% Usage Final Plate	
		SHAPE Based	NFP Based	SHAPE Based	NFP Based	SHAPE Based	NFP Based
20-F	52	230	239	4	4	6.4	20.0
21	4	22	2	1	1	20.3	20.3
22	27	198	30	1	1	47.3	47.3
23-G	84	449	320	3	3	47.9	55.7
24-H	56	432	1296	5	6	61.9*	11.7
25	38	183	376	2	2	59.3	52.4
26	3	10	4	1	1	33.7	33.7
27	26	359	46	2	2	11.7	5.2
28	76	433	297	3	3	28.0	26.6
29-I	107	790	1629	9	10	62.3*	38.0
30-J	111	1065	3691	8	8	6.5	17.7
31	7	63	211	1	1	48.6	48.6
32-K	37	175	147	2	2	13.5	21.0
33	79	803	245	2	2	44.4	43.4
34-L	52	422	394	5	5	58.2	64.9
35	44	244	219	5	5	12.8	15.9
36	12	74	8	2	2	14.8	14.8
37-M	68	515	586	3	3	2.9	11.6
38	101	1167	1788	3	3	12.2	15.7
39	2	4	1	1	1	4.3	4.3
40	36	249	914	5	5	11.0	8.5
41	23	114	334	3	3	40.5	37.5
42	4	14	6	1	1	26.6	26.6

* Required one less plate than other corresponding solution

In summary, the effectiveness of both the feature and NFP techniques varied over the broad set of problems studied. The performance observed depended in great part upon the characteristics of the profiles nested as well as the rating criteria used for analysis. The strengths and weaknesses of both approaches were evident in the layouts produced.

However, results generally indicate shorter computation times and better packing from the proposed shape reasoning approach. Additional commentary is deferred to the next chapter where concluding remarks and suggestions for further investigation are discussed.

Chapter 6
Conclusions and Future Work

6.1 Summary

The research presented in this manuscript confirms the fundamental premise that an effective solution to the irregular profile bin packing problem can be formulated using information about the configuration and shape of a layout's parts. This shape information is stored in the form of features, geometric constructs of two and three sides, which are extracted from both the unallocated part profiles and the region of the stock resource remaining available for use. A series of simplification techniques are applied to these entities, permitting the characterization of shape at the various scales of detail required for solution reasoning.

Layouts are generated in an iterative fashion. At each stage of nesting, all possible matches between the part and resource features are examined for the possible existence of complementary forms. Using the extracted shape information, a matching index is calculated to quantify the quality of fit achieved by each pair. Based on this measure, a set of intermediate allocations is generated using those parts most likely to produce efficient placements. The orientation and initial position of each of the chosen parts is provided by the designated align type, a description of the way in which two features mesh. A single layout for expansion in the next stage of the solution is then selected using a waste function which predicts the current and future trim loss associated with each arrangement.

The utility of the technique described has been demonstrated through testing on a diverse set of industry supplied problems. A comprehensive inspection of the feature based layouts generated indicates nests of comparable waste which are commonly superior to those of the NFP method. Individual nests within the feature base results will often exhibit exceptionally low wastes previously un-achieved using the NFP approach. However, the comparison of two techniques through the examination of individual plates is often misleading, as the parts found on each plate differ from solution to solution. Overall results from the complete set of resources used must also be considered.

Furthermore, no standard data sets exist on which to perform benchmark runs. Such issues complicate the appraisal of any nesting technique's merits. However, based on results from the total run of 42 problems, the relative merits of this technique are evident. On average, solutions requiring less stock material were produced at less computational expense.

6.2 Shape Reasoning Advantages and Limitations

An integral part of all packing problem solution algorithms is an ability to deal with the infinite number of placement possibilities occurring throughout the generation of a layout. Many researchers have coped with this task by limiting the available options at the outset. Although effective, this approach automatically restricts the number and type of solutions capable of being produced. The current method provides additional flexibility by avoiding the restrictions commonly used to yield computationally tractable solutions.

The most striking difference between the proposed method and previous reported techniques, is the ability to dynamically select optimal part orientations. This differs from the majority of techniques, such as the NFP, which restrict profiles to a distinct set of normally orthogonal rotations. Furthermore, at each stage of the solution the current formulation permits the selection and placement of any available part. This contrasts with other approaches which either allocate in a specific predetermined order or break the complete profile set into smaller more workable problems.

As is common with many heuristic methods, the documented shape reasoning approach mimics that of a human nester. The skilled eye of a technician can quickly scan a large set of profiles and easily detect the complementary shapes which exist between the parts and remaining resource. The success of the proposed algorithm stems from its ability to imitate this manual technique. Detection of the underlying profile shapes is made possible through elimination of unnecessary and confusing detail. With this accomplished all information can easily be stored in a basic feature format. Determination of the plausible part orientations and locations are then made quickly using matching indexes and align types. This avoids the brute force approach of the NFP technique, whereby a series of placements must be generated for all parts regardless of their merit. The finer details of parts are also retained however, and exploited during final placement. Here, the removal of any unnecessary space is made possible through the shifting and abutting of the complex original profiles.

One trend observed in both the NFP and feature based results was a noticeable increase in trim waste as solutions progressed from one plate to the next for a selected set of profiles. This can be attributed to the early depletion of smaller more easily placed parts which can fill gaps or recesses. For later plates what remains are usually large and highly irregular profiles. As parts become larger relative to the resource, much of the flexibility associated with feature based placement cannot be exploited, as the restrictions imposed by the boundary of the stock become more pronounced. As a concequence, a substantial improvement over previous methods for these later plates is sometimes not observed.

The efficient nesting of larger profiles often requires information about the future interaction of the parts to be allocated. Considerations must be made to combine *multiple* parts such that they conform to or exploit the dimension of the resourse involved. The iterative placement of *individual* parts does not permit this. At each stage of the current method, the next placement is resolved based upon interactions between single parts and the current configuration. Consequently predicting future effects is difficult. This limitation, however, is inherent in most iterative placment techniques. Possible avenues of improvement are suggested in the next section.

6.3 Future Work

In many cases the nests produced by the automated application can be improved through slight manual alteration, however the waste recovered by such operations is often minimal. Any gains made through manual interaction must be weighed against the increased costs in man hours. In industry, nesting involves the optimization of both material and labor used, as well as other issues including states of cutting equipment, stock availability and scheduling. If the advantages of manual layout were merited, productivity could be gained by incorporating the proposed techniques into a hybrid manual-automatic system. In such an application, feature matching heuristics would be employed to quickly provide a palate of suggested placements. Rather than depend upon a calculated waste function, the full experience and intuition of the technician could be exploited to select, and if necessary adjust the most promising layout for expansion. The capability to manually generate placements could also be integating into the application. Albano provides a detailed outline of such a pseudo automatic system [ALBA77].

Throughout the evolution of this research, a constant objective has been the development of a robust system capable of solving a diverse set of "real world" problems. Experiments have verified the stability of the algorithms implemented for a broad set of complex inputs. As with all system of this nature, tradeoffs must be made to accommodate the broad range of cases which can occur. Specializing for a specific set of characteristics is usually unwise, as such procedures invariably degrade algorithm performance on other sets. As a consequence, the resulting application is optimal for fewer problems although competent for all. From a practical viewpoint a dependable method is often preferable to a superior one which sometimes fails.

Beyond the need to solve a spectrum of problems, other factors prohibit the development of specialized optimum values for the tolerances and parameters controlling algorithm performance. As the simplest of industry problems often contain non-uniform distributions of profile characteristics, catering to the needs of even a single problem can be difficult. Opportunities for such improvements might exist however, if diverse and hard to manage problems could be transformed into sets with a narrower assortment of properties. This task might be accomplished through the clustering of complementary parts prior to nesting, assuming such matching profiles exist. With problems converted to a simpler form, investigation of optimal tolerance settings might be justified. Values for those parameters controlling part feature generation, remaining resource simplification and matching index calculations could be determined through comparative runs of the application.

Another issue deserving additional study is determination of the best successor for expansion. Although information from the set of remaining parts is used to dynamically control the waste function, it is still inherently limited to examining only the current solution. Unfortunately, the detrimental effect of certain placements can only be seen in successive steps. One solution to such problems is to adopt a less deterministic search algorithm which explores several potential paths at each stage of the solution process. Unfortunately, results from the backtracking technique implemented in the pilot study indicated a substantial increase in solution time without a consistent reduction in waste.

The potential benefits from a new search method must be weighed against the extra computational expense of evaluating any additional part placements required. If the present formulation of the waste function is to be used for such calculations, algorithms for determining the useful and un-usable areas of a layout must be optimized, as these currently account for 60 to 80 percent of run times. An alternative would be the use of a less rigorous waste function coupled with more exacting matching indexes. The additional information needed for new index formulations might be provided by further exploiting the geometry associated with each align type. Such an approach could be effective at eliminating less favorable placements.

The allocation of irregular shapes is a strongly domain dependent problem. As such, it is difficult if not impossible to develop generic solution strategies which are optimal when applied to all cases. To varying degrees, expert systems overcome this issue by addressing only specific problem types. These solutions are effective but limited in application. Unfortunately in many industry no single approach is appropriate for the full range of problems encountered. For this reason, the methods presented throughout this document have attempted to deal with profile allocation at the most fundamental of levels, using the individual shape of parts. However, by ignoring the overall character of the problem studied, potential clues to the best method may be lost. In the future, intelligent systems will qualify the nature of a problem during processing, permitting application of the most appropriate method available. The shape reasoning heuristics developed will provide useful tools for accomplishing this task, adding to the arsenal of methods available for solving this complex problem.

Bibliography

[ADAM76] Adamowicz, M., Albano, A. "A Solution to the Rectangular Cutting-Stock Problem", *IEEE Transactions on Systems, Man & Cybernetics*, April 1976, pp. 302-310.

[ADAM79] Adamowicz, M., Albano, A. "Nesting Two-dimensional Shapes in Rectangular Modules", *Computer Aided Design*, No. 2, Vol. 8, April 1979, pp. 27-33.

[ALBA77] Albano, A., "A Method to Improve Two-dimensional Layout", *Computer Aided Design*, Vol. 9, No. 1, January 1977, pp. 48-52.

[ALBA80] Albano, A., Sapuppo, Giuseppe, "Optimal Allocation of Two-Dimensinal Irregular Shapes Using Heuristic Search Methods", *IEEE Transactions on Systems, Man & Cybernetics*, May 1980, pp. 242-248.

[ARBE93] Arbel, Ami, "Large -Scale Optimization Methods Applied to the Cutting Stock Problem of Irregular Shapes", *International Journal of Production Research,* Vol. 31, No. 2, 1993, pp. 483-500.

[BENG82] Bengtsson, Bengt-Erik, "Packing Rectangular Pieces - A Heuristic Approach", *The Computer Journal*, Vol. 25, No. 3, 1982, pp. 353-357.

[BOWY93] Bowyer, Adrian, & Woodwark, John, *Introduction to Computing with Geometry*, Information Geometers Ltd., Winchester UK, 1993.

[BRON83] Bronsoiler, A., Sanders, J.L., "Performance Testing of Irregular Parts Nesting Systems for Flame Cutting and Other Industrial Applications", *1983-SME Manufacturing Engineering Transactions 11th NAMRC North American Manufacturing Research Conference Proceedings*, pp. 434-440.

[CAME86] Cameron, S.A., Culley, R. K., "Determining the Minimum Translational Distance Between Two Convex Polyhedra", CH2282-2/86/0000/0591 1986 IEEE, pp. 591-596.

[CAVI89] Caviglia, Daniele D., Bisio, Giacomo M., Curatelli, Francesco, Giovannacci, Luca, & Raffo, Luigi, "Neural Algorithms for Cell Placement in VLSI Design", *Proceedings of International Joint Conference on Neural Networks,* 1989, IEEE Catalog Number 89CH2765-6, pp. I573 - I580.

[CHEO91] Cheok, B.T., and Nee, Y.C., "Algorithms for Nesting of Ship/Offshore Structural Plates", *Advances in Design Automation -1991-*, DE-Vol. 32.-2, ASME, pp. 221-226.

[CHRI77] Christofides, Nicos, & Whitlock, Charles, "An Algorithm for Two-Dimensional Cutting Problems", *Operations Research,* Vol. 25, No. 1, January-February 1977, pp. 30-44.

[CHUN89] Chung, Jason, & Hillman, Danald J., "Object Oriented Nesting System on Two-dimentional Highly Irregular Resoruces", *Intelligent Robots and Computer Vision VIII: Systems and Applications*, SPIE Vol. 1193, 1989, pp. 54-63.

[COFF90] Coffman, E.G., & Shor, P. W., "Average-case Analysis of Cutting and Packing in Two Dimensions", *European Journal of Operational Research,* Vol. 44, No. 2, January 25, 1990, pp. 134-144.

[DAGL83] Dagli, Cihan H., & Totoglu, M. Yalcin, "A Computer Package for Solving Cutting Stock Problems", *7th International Conference on Production Research,* Windsor, Ontario, Canada, Vol. 1, 1983, pp. 480-486.

[DAGL87] Dagli, Cihan, & Tatoglu, M. Yalcin, "An approach to Two-Dimensional Cutting Stock Problems", *International Journal of Production Research,* Vol. 25, No. 2, 1987, pp. 175-190.

[DAGL88] Dagli, Cihan, "Cutting Stock Problem: Combined Use of Heuristics and Optimization Methods", *Recent Developments in Production Research*, A. Mital editor, Elsevier Science Publishers B.V., Amsterdam, 1988, pp. 500-506.

[DAGL90a] Dagli, Cihan, Hajakbari, Alireza, "Parallel Architecture for Stock Cutting Problem", PARABASE 90 Int Conf Databases Parallel Archit Appl PARABASE 90: International Conference on Databases, Parallel Architectures, and Their Applications, Miami Beach, FL, USA, Mar 7-9 1990 (IEEE cat n 90CH2728-4) CH2728-4/90/0000/0534 1990 IEEE, p. 534.

[DAGL90b] Dagli, Cihan H., & Hajakbari, Alireza, "Simulated Annealing Approach for Solving Stock Cutting Problems", *Proceedings of IEEE International Conference on Systems, Man, and Cybernetics,* 1990, IEEE Catalog Number 90CH2930-6, pp. 221-223.

[DAVI77] Davis, Larry S., "Understanding Shape: Angles and Sides", *IEEE Transactions on Computers*, Vol. c-26, No. 3, March 1977, pp. 236-242.

[DECA78] DeCani, Phillip, "A Note on the Two-dimensional Rectangular Cutting-stock Problem", *Journal of the Operational Research Society*, Vol. 29, No. 7, 1978, pp. 703-706.

[DIET91] Dietrich, Robert D., & Yakowitz, Sidney J., "A Rule-Based Approach to the Trim-Loss Problem", *International Journal of Production Research,* Vol. 29, No. 2, 1991, pp. 401-415.

[DYCK90] Dyckhoff, Harald, "A Typology of Cutting and Packing Problems", *European Journal of Operational Research*, Vol. 44, No. 2, January 25, 1990, pp. 145-159.

[DYCK92] Dyckhoff, Harald, & Finke, Ute, *Cutting and Packing in Production and Distribution A Typology and Bibliography,* Physica-Verlag, A Springer-Verlag Company, Heidelberg, 1992.

[FARL90] Farley, Alan A., "The Cutting Stock Problem in the Canvas Industry", *European Journal of Operational Research*, Vol. 44, No. 2, January 25, 1990, pp. 247-255.

[FOLE84] Foley, James D., & Van Dam, Andries, *Fundmentals of Interactive Compter Graphics*, Addison Wesley Publishing Company, Reading, Massachusetts, 1984.

[FREE64] Freeman, H., & Garder, L., "Apictorial Jigsaw Puzzles: The Computer Solution of a Problem in Pattern Recognition", *IEEE Transactions on Electronic Computers*, Vol. 13, 1964, pp. 118-127.

[FREE74] Freeman, Herbert, "Computer Processing of Line-Drawing Images", *Computing Surveys*, Vol. 6, No. 1, March 1974, pp. 57-97.

[FREE75a] Freeman, H., & Shapira, R., "Determining the Minimum-Area Encasing Rectangle for an Arbitrary Closed Curve", *Communications of the ACM*, Vol. 18, No. 7, July 1975, pp. 409-413.

[FREE75b] Freeman, Herbert, "On the packing of Arbitrary-Shaped Templates", *Second USA-JAPAN Computer Conference*, 1975, pp. 102-107.

[FREE78] Freeman, Herbert, "Shape Description Via the Use of Critical Points", *Patern Recognition*, Vol. 10, 1978, pp. 159-166.

[FUJI93] Fujita, Kikuo, Akagi, Shinsuke, & Hirokawa, Noriyasu, "Hybrid Approach for Optimal Nesting Using a Genetic Algorithm and a Local Minimization Algorithm", *Advances in Design Automation*, Vol. 1, ASME 1993, pp. 477-484.

[GANT93] Ganter, M.A., & Skoglund, P.A., "Feature Extraction for Casting Core Development", *ASME Journal of Mechanical Design*, Vol. 115, December 1993, pp. 744-750.

[GARE79] Garey, Michael R., & Johnson, David S., *Computers and Intractability*, W.H. Freeman and Company, San Francisco, 1979.

[GEMM92] Gemmill, Douglas D., "Solution to the Assortment Problem Via the Genetic Algorithm", *Mathematical and Computer Modeling*, Vol. 16, No. 1, 1992, pp. 89-94.

[GILL65] Gilmore, P.C., & Gomory, R.E., "Multi-Stage Cutting Stock Problems of Two and More Dimensions", *Operations Research*, Vol. 13, 1965, pp. 94-120.

[GONZ78] Gonzalez, Rafael C., & Thomason, Michael G., *Syntactic Pattern Recognition an Introduction*, Addison-Wesley, London, 1978. Chapter 6, Pavlidis, T., & Feng, H.Y.F., "Shape Discrimination", pp 125-144.

[HAES91] Haessler, Robert W., & Sweeney, Paul E., "Cutting Stock Problems and Solution Procedures", *European Journal of Operational Research*, Vol. 54, 1991, pp. 141-150.

[HAHN68] Hahn, Susan G., "On the Optimal Cutting of Defective Sheets", *Operations Research*, Vol. 16, No. 8, 1968, pp. 1100-1114.

[HAIM70] Haims, Murray J., Freeman, Herbert, "A Multistage Solution to the Template-Layout Problem", *IEEE Transactions on Systems Science & Cybernetics*, April 1970, pp. 145-151.

[HART85] Hartigan, P.M., "Computation of the Dip Statistict Test for Unimodality", *Applied Statistics*, Vol. 34, 1985, pp. 320-325.

[HILL90] Hill, Fransis S., *Computer Graphics*, Macmillan Publishing Company, New York, 1990.

[HINX80] Hinxman, A.I., "The Trim-loss and Assorment Problems: A Survey", *European Journal of Operational Research*, 5 (1980), pp. 8-18.

[HORO75] Horowitz, Steven L., "A Syntactic Algorithm for Peak Detection in Waveforms with Applications to Cardiography", *Communications of the ACM*, Vol. 18, No. 5, May 1975, pp. 281-285.

[ILLI89] Illiev, V.I., Popov, G.S., Ivanov, M., "A System for Computer-aided Technical Preparation in the Production of Plane Parts by Stamping", *Journal of Mechanical Working Technology*, 18 (1989), pp. 283-292.

[ISRA82] Israni, Sharat, Sanders, Jerry, "Two-dimensional Cutting Stock Problem Research: a Review and a New Rectangular Layout Algorithm", *Journal of Manufacturing Systems*, Vol. 1, No. 2, pp. 169-182.

[ISRA85] Israni, Sharat S., Sanders, Jerry L. , "Performance Testing of Rectangular Parts-nesting Heuristics", *International Journal of Production Research*, 1985, Vol. 23, No. 3, pp. 437-456.

[JAIN92] Jain, P., Fenyes, P., Richter R., "Optimal Blank Nesting Using Simulated Annealing", *Journal of Mechanical Design* (Transactions of the ASME), March 1992, Vol. 114, pp. 160-165.

[KAMP88] Kampke, Thomas, "Simulated Annealing: Use of a New Tool in Bin Packing", *Annals of Operations Research*, Vol 16., 1988, pp. 327-332.

[KIRK83] Kirkpatrick, S., Gelatt, C.D., & Vecchi, M.P., "Optimization by Simulated Annealing", *Science*, Vol. 220, No. 4598, May 13, 1983, pp. 671-680.

[LAMO86] Lamousin, H.J., Waggenspack, W.N., & Dobson, G.T. "Nesting of Complex 2-D Parts Within Irregular Boundaries", *Journal of Manufacturing Science and Engineering*, August 6, 1986

[LOZA83] Lozano-Perez, Tomas, "Spatial Planning: A Configuration Space Approach", *IEEE Transactions on Computers*, Vol. C-32, No. 2, February 1983, pp. 108-120.

[LUTF92] Lutfiyya, Hanan, McMillin, Bruce, Poshyananda, Pipatpong, & Dagli, Cihan, "Composite Stock Cutting Through Simulated Annealing", *Mathematical and Computer Modeling,* Vol. 16, No. 1, 1992, pp. 57-74.

[NEE86] Nee, A.Y.C., Seow, K.W., Long, S.L. "Designing Algorithm for Nesting Irregular Shapes With and Without Boundary Constraints", *Annals of the CIRP*, Vol. 35/1/1986, pp. 107-110.

[NEE88] Nee, A.Y.C., Long, S.L., "Microcomputer-aided Material Management in a Furniture Factory", *Computers in Industry*, 10 (1988), pp. 221-229.

[NIEV82] Nievergelt, J, Preparata, F.P., "Plane-Sweep Algorithms for Intersectin Geometric Figures", *Communications of the ACM*, October 1982, Vol. 25, Number 10, pp. 739-747.

[NOAK91] Noaker, Paula, "Software that Packs a Punch. CAD/CAM tools allow fabrication parts just in time", *Manufacturing Engineering*, Vol. 107, No. 5, November 1991, pp. 93-96.

[OLIV90] Oliveira, Jose Fernando, Ferreira, Jose Soeiro, "An Improved Version of Wang's Algorithm for Two-Dimensional Cutting Problems", *European Journal of Operational Research*, Vol. 44, No. 2, January 25, 1990, pp. 256-266.

[PAVL73] Pavlidis, Theodosios, "Waveform Segmentation Through Functional Approximation", *IEEE Transactions on Computers*, Vol. c-22, No. 7, July 1973, pp. 689-697.

[PAVL74] Pavlidis, Theodosios, & Horowitz, Steven L., "Segmentation of Plane Curves", *IEEE Transactions on Computers*, Vol. c-23, No. 8, August 1974, pp. 860-870.

[PAVL78] Pavlidis, Theodosios, "Survey A Review of Algorithms for Shape Analysis", *Computer Graphics and Image Processing*, Vol. 7, 1978, pp. 243-258.

[PAVL80] Pavlidis, T., *Structural Pattern Recognition, Springer Verlag Series in Electrophysics,* Springer Verlag, Berlin, Heidleberg, New York, 1980.

[POSH90] Poshyanonda, Pipatpong, Bahrami, Ali, & Dagli, Cihan H., "Two Dimensional Nesting Problem: Artificial Neural Network and Optimization Approach", *Proceedings of 1990 IEEE International Confernce on Systems, Man, and Cybernetics*, November 4-7, 1990, IEEE Catalog Number 90CH2930-6, ISBN 0-87942-597-0, pp. IV 572 - IV 577.

[PRAS91] Prasad, Y.K.D.V., Somasundaram, S. "CASNS - A Heuistic Algorithm for the Nesting of Irregular-Shaped Sheet Metal Blanks", *Computer Aided Engineering Journal*, Vol. 8, No. 2, April 1991, pp. 69-73.

[PRAS94] Prasad, Y.K.D.V., "A Set of Heuristic Algorithms for Optimal Nesting of Two-Dimensional Irregularly Shaped Sheet-Metal Blanks", *Computers in Industry,* 24 (1994), pp. 55-70.

[PREP85] Preparata, Franco P., & Shamos, Michael Ian, *Computational Geometry an Introduction,* Springer-Verlag, New York, Berlin, 1985.

[QU87] Qu, Weishuang, Sanders, Jerry, "A Nesting Algorithm for Irregular Parts and Factors Affecting Trim Losses", *Interanational Journal of Production Research,* 1987, Vol. 25, No. 3, pp. 381-397.

[QU89] Qu, Weishuang, Sanders, Jerry L., "Sequence Selection of Stock in Two-Dimensional Layout Problems", *International Journal of Production Research,* 1989, Vol. 27, No. 9, pp. 1553-1571.

[RADA82] Radack, Gerald M. & Badler, Norman I., "Jigsaw Puzzle Matching Using A Boundary-Centered Polar Encoding", *Computer Graphics and Image Processing,* Vol. 19, 1982, pp. 1-17.

[RINN87] Rinnooy Kan, A.H.G., DeWit, J.R., and Wijmenga, R.Th., "Nonorthogonal Two-Dimensional Cutting Patterns", *Management Science,* Vol. 33, No. 5, May 1987, pp. 670-684.

[ROSE73] Rosenfeld, Azriel, & Johnston, Emily, "Angle Detection on Digital Curves", *IEEE Transactions on Computers,* September 1973, pp. 875-878.

[ROSE75] Rosenfeld, Azriel, & Weszka, Joan, "An Improved Method of Angle Detection on Digital Curves", *IEEE Transactions on Computers,* September 1975, pp 940-941.

[SAND88] Sandgren, E., Dworak, T., "Part Layout Optimization using a Quadtree Representation", *Proceedings of the 1988 Design Automation Conference,* Kissimmee, Florida, pp. 211-219.

[SARI83] Sarin, S.C., "Two-Dimensional Stock Cutting Problems and Solutions Methodologies", *Journal of Engineering for Industry* (Trans. of the ASME), August 1983, Vol. 105, pp. 155-160.

[SCHA78] Schachter, Bruce, "Decomposition of Polygons into Convex Sets", *IEEE Transactions on Computers,* Vol. C-27, No. 11, November 1978, pp. 1078-1082.

[SERR82] Serra, J.Q. *Image Analysis and Mathematic Morphology,* Adison Wesley, New York, 1982.

[SHAH95] Shah, Jami, & Mantyla, Martti, *Parametric and Feature Based CAD/CAM Concepts, Techniques, and Application,* John Wiley & Sons Inc., New York, 1995.

[SHAM76] Shamos, Michael I., & Hoey, Dan, "Geometric Intersection Problems", *17th Annual Symposium on Foundations of Computer Science*, Houston, 1976, pp. 208-215.

[SPER79] Sperling, B. "Nesting Is More Than a Layout Problem", *Computer Applications in the Automation of Shipyard Operation and Ship Design III*, 1979, pp. 287-294.

[STAL83] Staley, S. M., Henderson, M.R., & Anderson, D. C., "Using Syntactic Pattern Recognition to Extract Feature Information From a Solid Geometric Data Base", *Computers in Mechanical Engineering*, September 1983, pp. 61-66.

[SUMM87] Summer, Charles J., & McNamara, James N., "Power Considerations for the Dip Test of Unimodality Using Mixtures of Normal and Uniform Distributions", *American Statistical Association 1987 Proceedings of the Statistical Computing Section,* 1987, pp. 186-191.

[SZYK95] Szykman, S. & Cagan, J., "A Simulated Annealing-Based Approach to Three-Dimensional Component Packing", *Journal of Mechanical Design,* June 1995, Vol. 117, pp. 308-314.

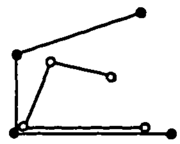
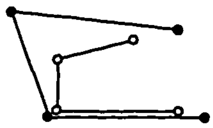
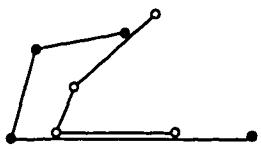
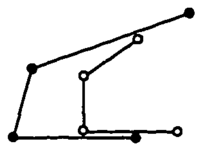
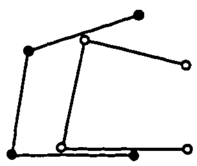
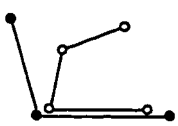
[YUZU87] Yuzu, Cai, Lujun, Liu, Wangwei, "An Expert System for Automatic Allocation of 2D Irregular Shapes", *Proceedings of Expert Systems in Computer Aided Design Conference,* International Federation for Information Processing 1987, pp. 407-423.

Appendix

A.1 Align Types

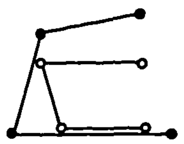
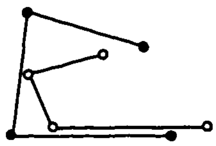
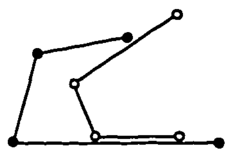
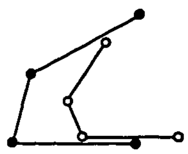
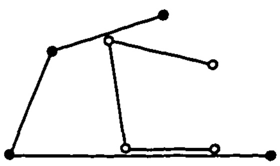
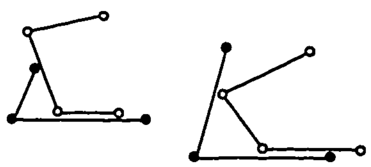
Each of the supported align types is shown in tables A.1.1 through A.1.3. For those containing an extra degree of freedom along the primary side, an arrow symbol ($\Leftarrow$, $\Rightarrow$) is included to indicate the desired direction of shift and the secondary side of the match. All other nomenclature not specifically noted is referenced from section 3.6 and Figure 3.13. Further details concerning align types are found in section 4.4.2.

Table A.1.1 Align Types for Side One Alignment

Align Type	$(\theta_{1P} < \theta_{1V})$	Class	Reference Vertex	Reference Edge	Other Conditions
1A		T	V_{2P}	E_{1V}	$\theta_{BP} < \theta_{BV}$
1B		T	V_{2P}	E_{1V}	$\theta_{BP} > \theta_{BV}$
1C		NT	V_{2P}	E_{1V}	
1D		NT	V_{2P}	E_{1V}	
1E		NT	V_{2P}	E_{1V}	$\theta_{BV} > 0$
1F		T	V_{2P}	E_{1V}	$l_{3V} = 0$

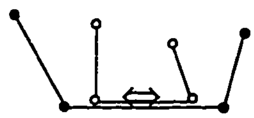
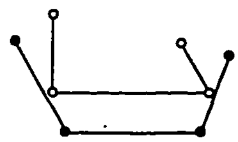
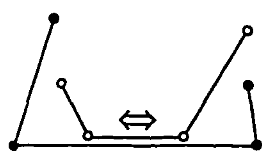
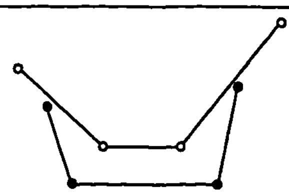
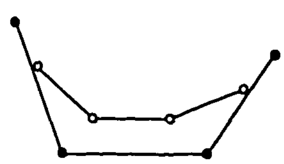
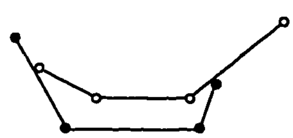
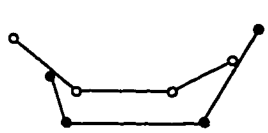
"T" = Touching "NT" = Non Touching

(table con'd.)

Align Type	$(\theta_{1P} > \theta_{1V})$	Class	Reference Vertex	Reference Edge	Other Conditions
1G		T	V_{2P}	E_{1V}	$\theta_{BP} < \theta_{BV}$
1H		T	V_{2P}	E_{1V}	$\theta_{BP} > \theta_{BV}$
1I		NT	V_{2P}	E_{1V}	
1J		NT	V_{2P}	E_{1V}	
1K		NT	V_{2P}	E_{1V}	$\theta_{BV} > 0$
1L		T	V_{2P}	E_{1V}	$l_{3V} = 0$

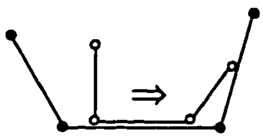
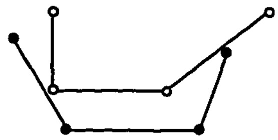
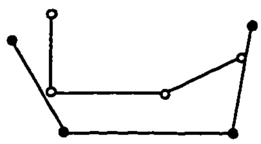
"T" = Touching "NT" = Non Touching

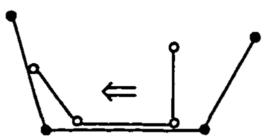
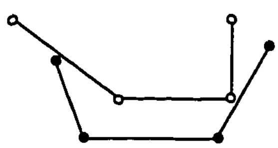
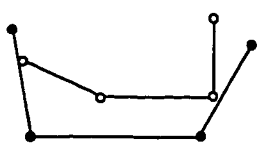
Table A.1.2 Align Types for Side Two Alignment

Align Type	$(\theta_{1P} < \theta_{1V})$ *AND* $(\theta_{2P} < \theta_{2V})$	Class	Reference Vertex	Reference Edge	Other Conditions
2A		T	V_{2P} *or* V_{3P}	E_{2V}	
2B		NT	V_{2P}	E_{1V}	$\theta_{BV} > 0$
Align Type	$(\theta_{1P} > \theta_{1V})$ *AND* $(\theta_{2P} > \theta_{2V})$	**Class**	**Reference Vertex**	**Reference Edge**	**Other Conditions**
2C		T	V_{2P} *or* V_{3P}	E_{2V}	
2D		NT	V_{1V}	E_{1V}	
2E		NT	V_{1P}	E_{1V}	$\theta_{BV} > 0$
2F		NT	V_{4P}	E_{3V}	
2G		NT	V_{1P}	E_{1V}	

"T" = Touching "NT" = Non Touching

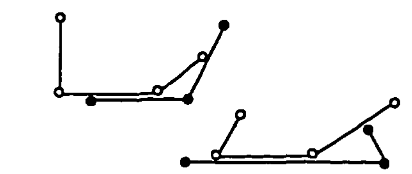
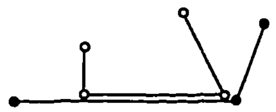
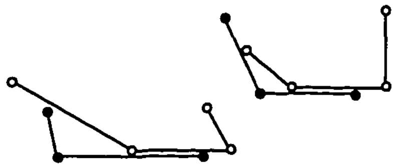
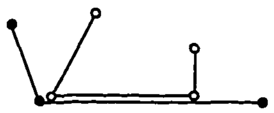
(table con'd.)

Align Type	$(\theta_{1P} > \theta_{1V})$ *AND* $(\theta_{2P} < \theta_{2V})$	Class	Reference Vertex	Reference Edge	Other Conditions
2H		T	V_{1P}	E_{2V}	
2I		NT	V_{3P}	E_{3V}	
2J		NT	V_{1P}	E_{1V}	$\theta_{BV} > 0$

Align Type	$(\theta_{1P} < \theta_{1V})$ *AND* $(\theta_{2P} > \theta_{2V})$	Class	Reference Vertex	Reference Edge	Other Conditions
2K		T	V_{3P}	E_{2V}	
2L		NT	V_{2P}	E_{1V}	
2M		NT	V_{2P}	E_{1V}	$\theta_{BV} > 0$

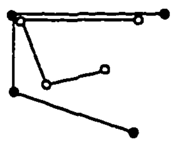
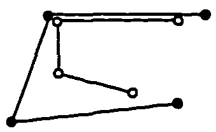
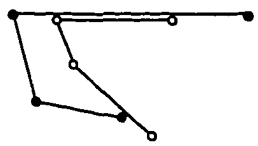
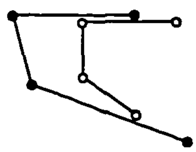
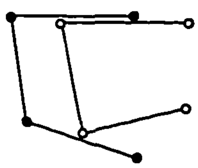
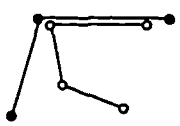
"T" = Touching "NT" = Non Touching

(table con'd.)

Align Type	$(\theta_{1P} < \theta_{1V})$	Class	Reference Vertex	Reference Edge	Other Conditions
2N		T	V_{2P}	E_{2V}	$l_{3V} = 0$
	$(\theta_{1P} < \theta_{1V})$				
2O		T	V_{2P}	E_{2V}	$l_{3V} = 0$
	$(\theta_{2P} < \theta_{2V})$				
2P		T	V_{3P}	E_{2V}	$l_{1V} = 0$
	$(\theta_{2P} < \theta_{2V})$				
2Q		T	V_{3P}	E_{2V}	$l_{1V} = 0$

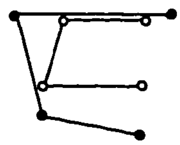
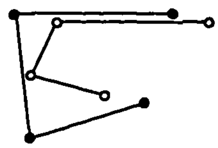
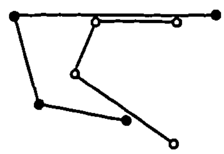
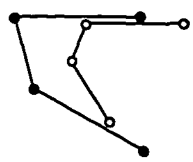
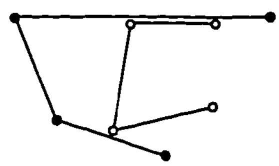
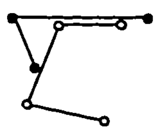
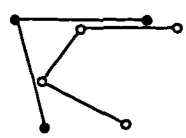
"T" = Touching "NT" = Non Touching

Table A.1.3 Align Types for Side Three Alignment

Align Type	$(\theta_{2P} < \theta_{2V})$	Class	Reference Vertex	Reference Edge	Other Conditions
3A		T	V_{3P}	E_{3V}	$\theta_{BP} < \theta_{BV}$
3B		T	V_{3P}	E_{3V}	$\theta_{BP} > \theta_{BV}$
3C		NT	V_{3P}	E_{3V}	
3D		NT	V_{3P}	E_{3V}	
3E		NT	V_{3P}	E_{3V}	$\theta_{BV} > 0$
3F		T	V_{3P}	E_{3V}	$l_{1V} = 0$

"T" = Touching "NT" = Non Touching

(table con'd.)

Align Type	$(\theta_{2P} > \theta_{2V})$	Class	Reference Vertex	Reference Edge	Other Conditions
3G		T	V_{3P}	E_{3V}	$\theta_{BP} < \theta_{BV}$
3H		T	V_{3P}	E_{3V}	$\theta_{BP} > \theta_{BV}$
3I		NT	V_{3P}	E_{3V}	
3J		NT	V_{3P}	E_{3V}	
3K		NT	V_{3P}	E_{3V}	$\theta_{BV} > 0$
3L	 	T	V_{3P}	E_{3V}	$l_{1V} = 0$

"T" = Touching "NT" = Non Touching

A.2 A Sample Problem Description

Figures A.2.1 through A.2.4 show a sample of the detailed problem description output by the feature based application. The example depicted is for Problem I. Individual values within the distribution are indicated by the black dots contained in each histogram bar. Futher information concerning the subjects for each figure may be found in section 5.5.

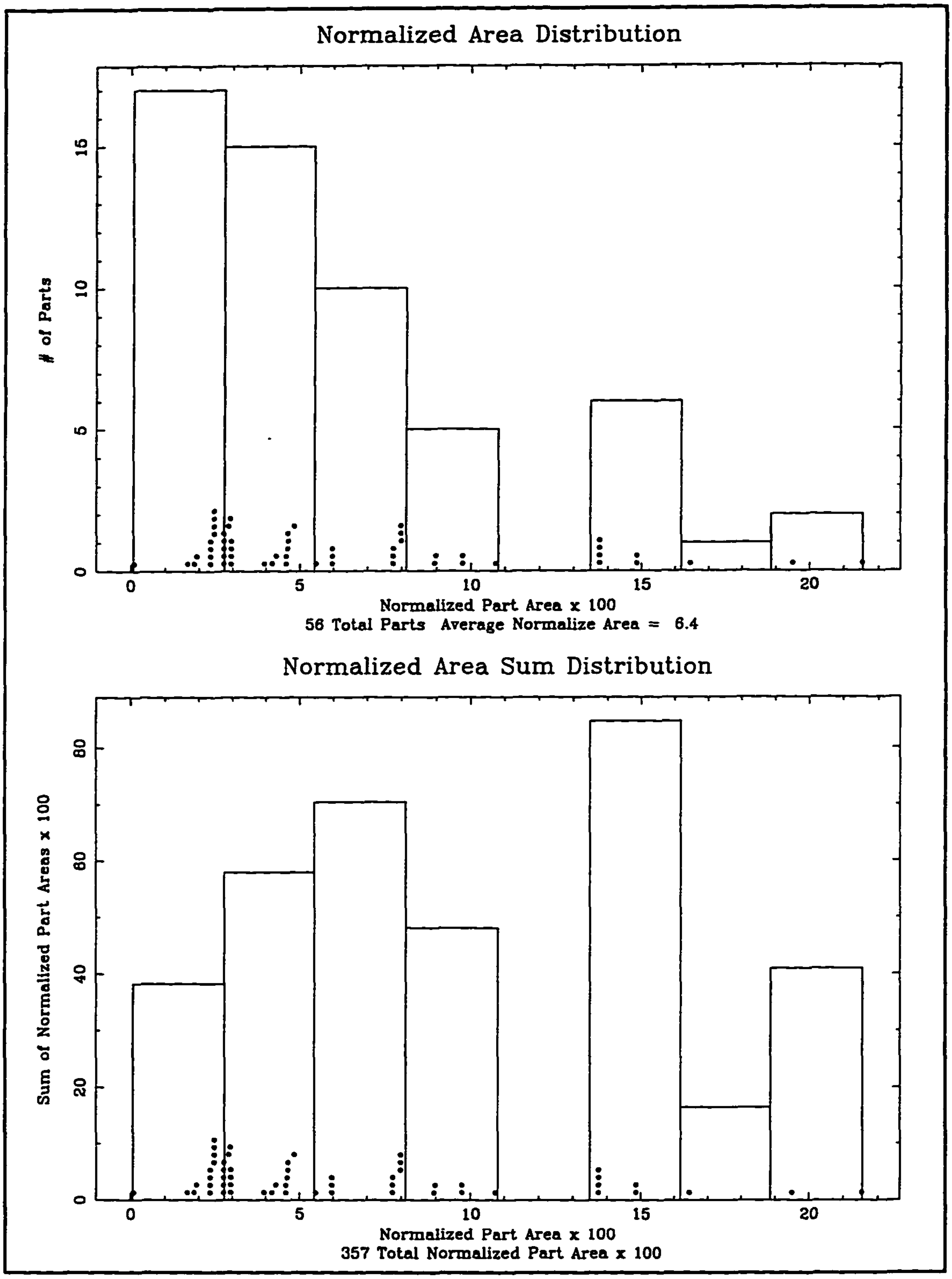


Figure A.2.1 The Normalized Area and Normalized Area Sum histograms. The height of each bar in the lower graph represents the sum of area values from corresponding range in the distribution above.

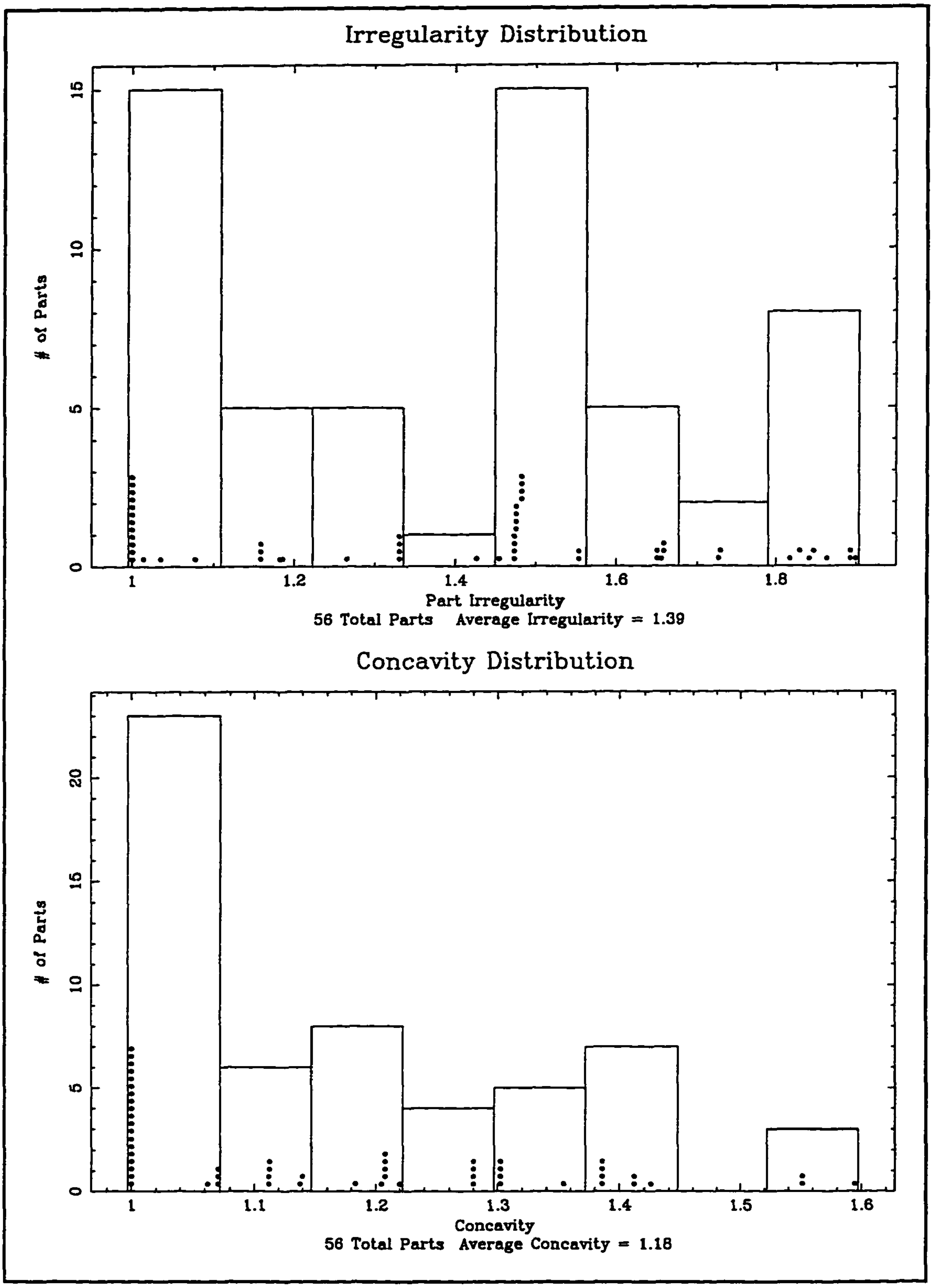


Figure A.2.2 The Part Profile Irregularity and Concavity histograms

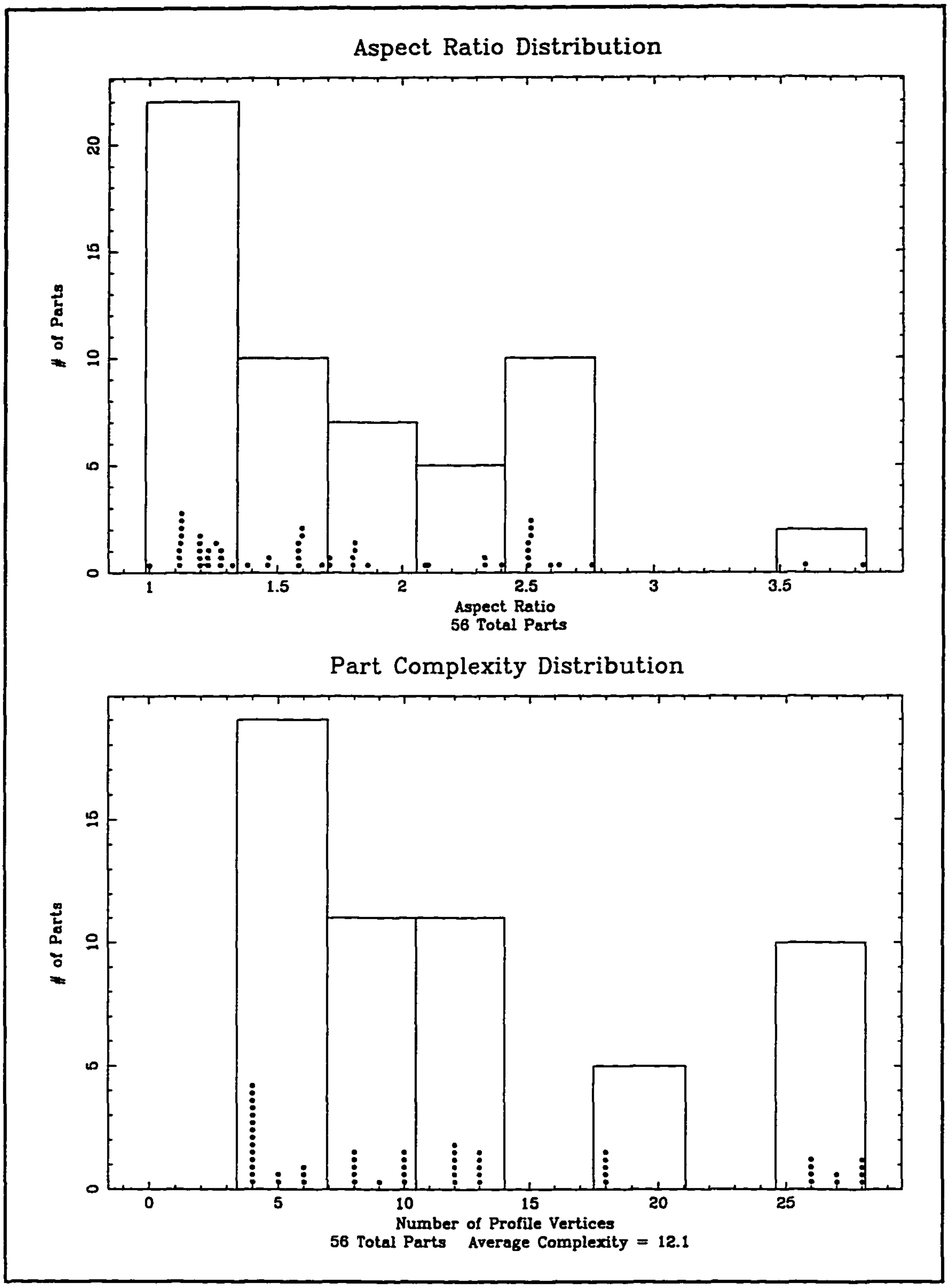


Figure A.2.3 The Aspect Ratio and Part Complexity histograms

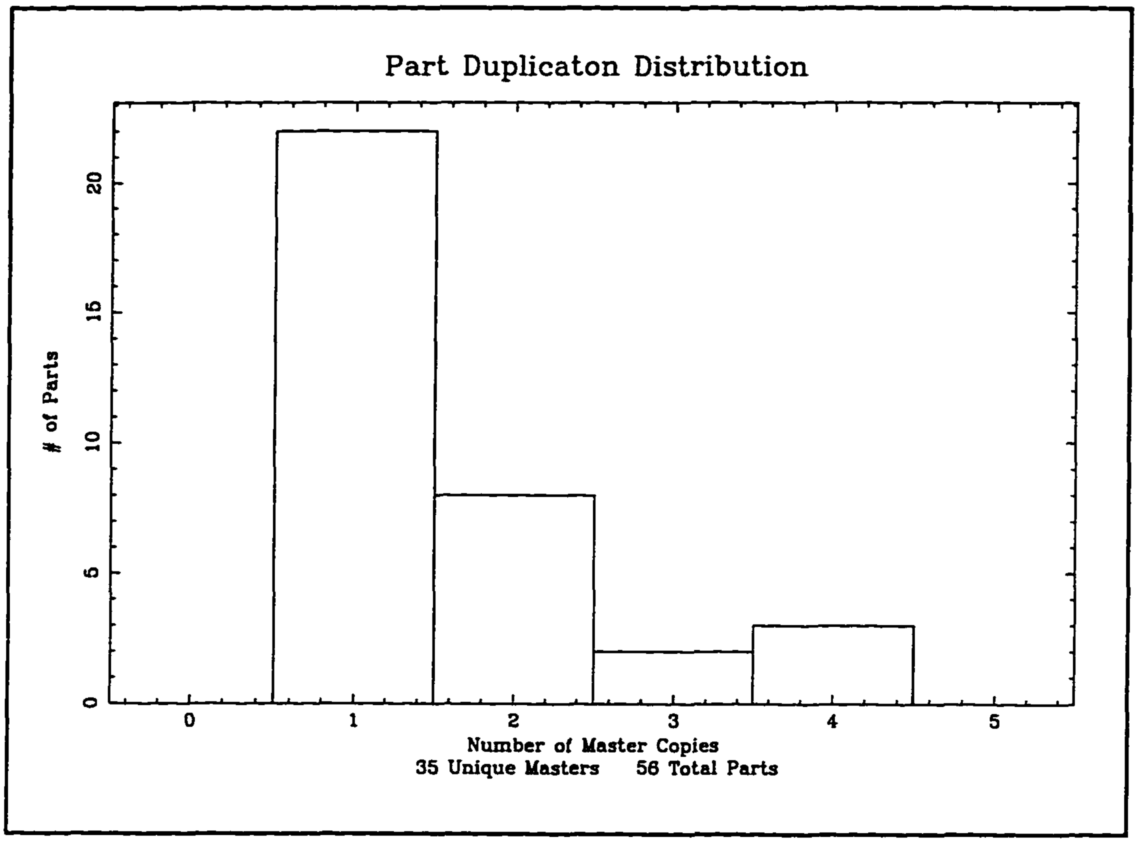


Figure A.2.4 The Part Duplication histogram. The height of each bar indicates the number of parts associated with the multiplicity shown on the horizontal axis.

A.3 Feature Based Solution Layouts

Figures A.3.1 through A.3.13 show feature based solutions for the problems of the parameter study. Layouts are plotted in the order of their creation from left to right across each row. N_SUCCESSOR is 20 with border restrictions alternating between 90°→180° and 180°→270°. Additional information for each problem concerning the profile characteristics, CPU times, and trim waste can be found in Tables 5.2, 5.4, and 5.5.

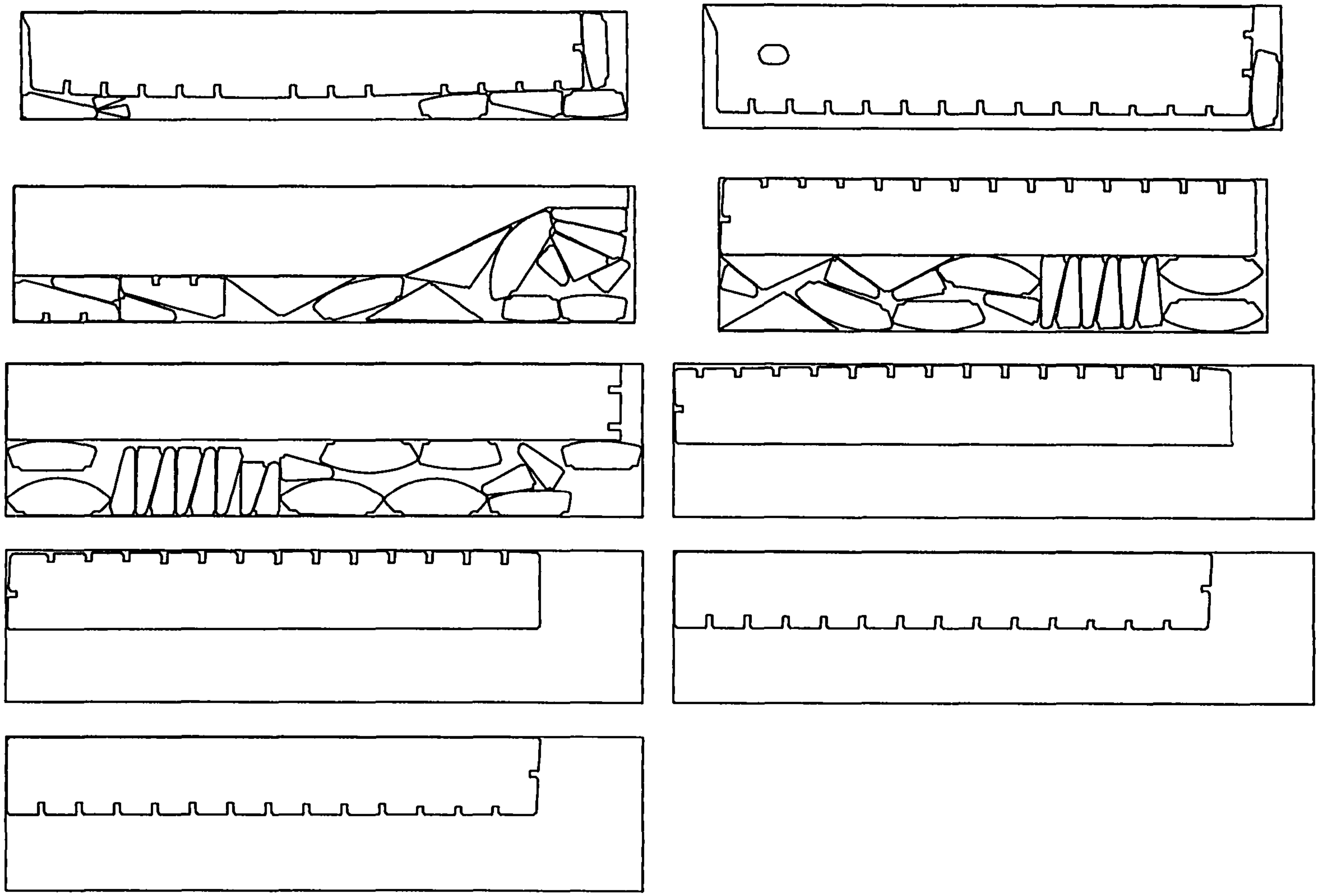


Figure A.3.1 Problem A

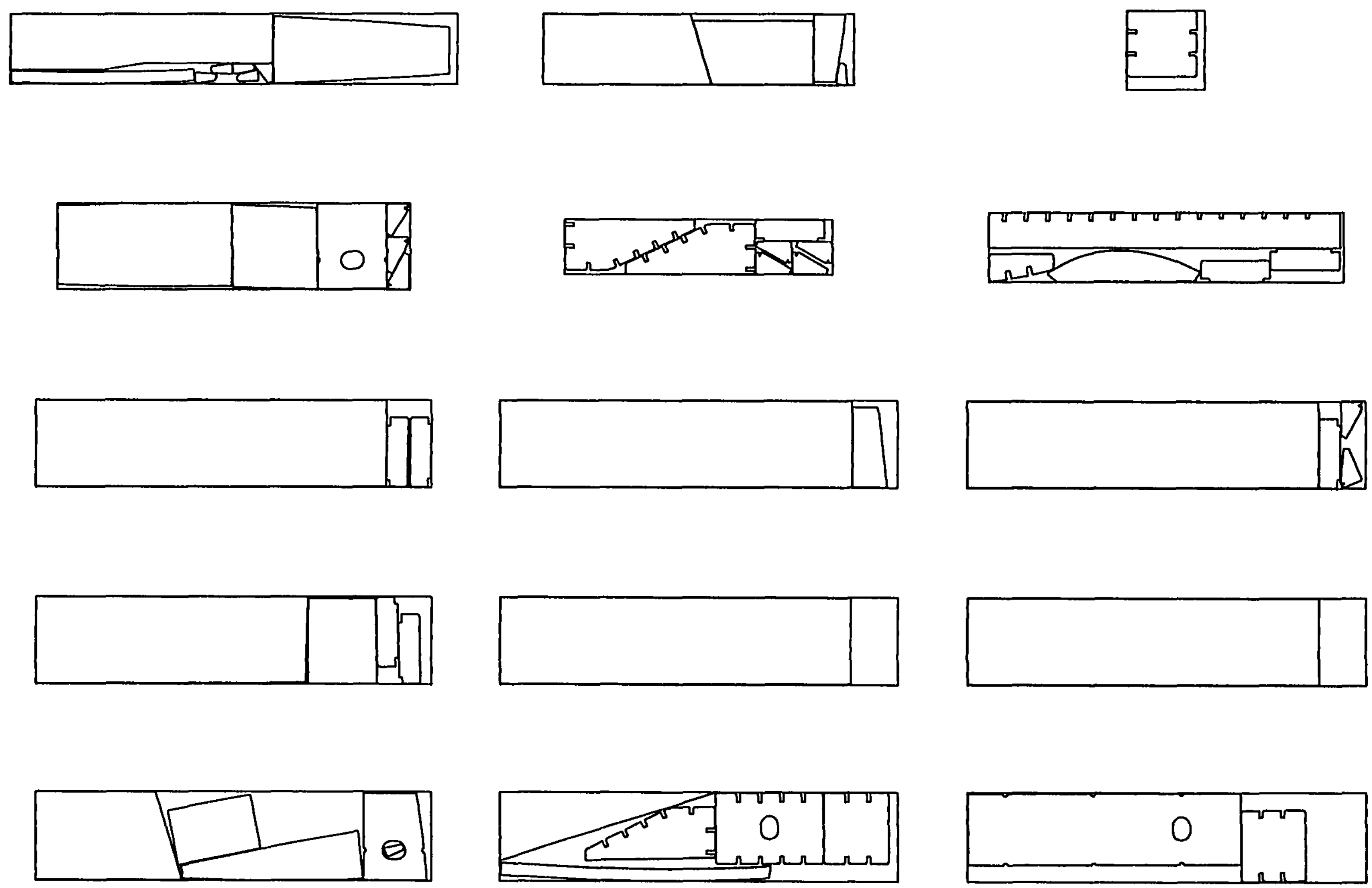
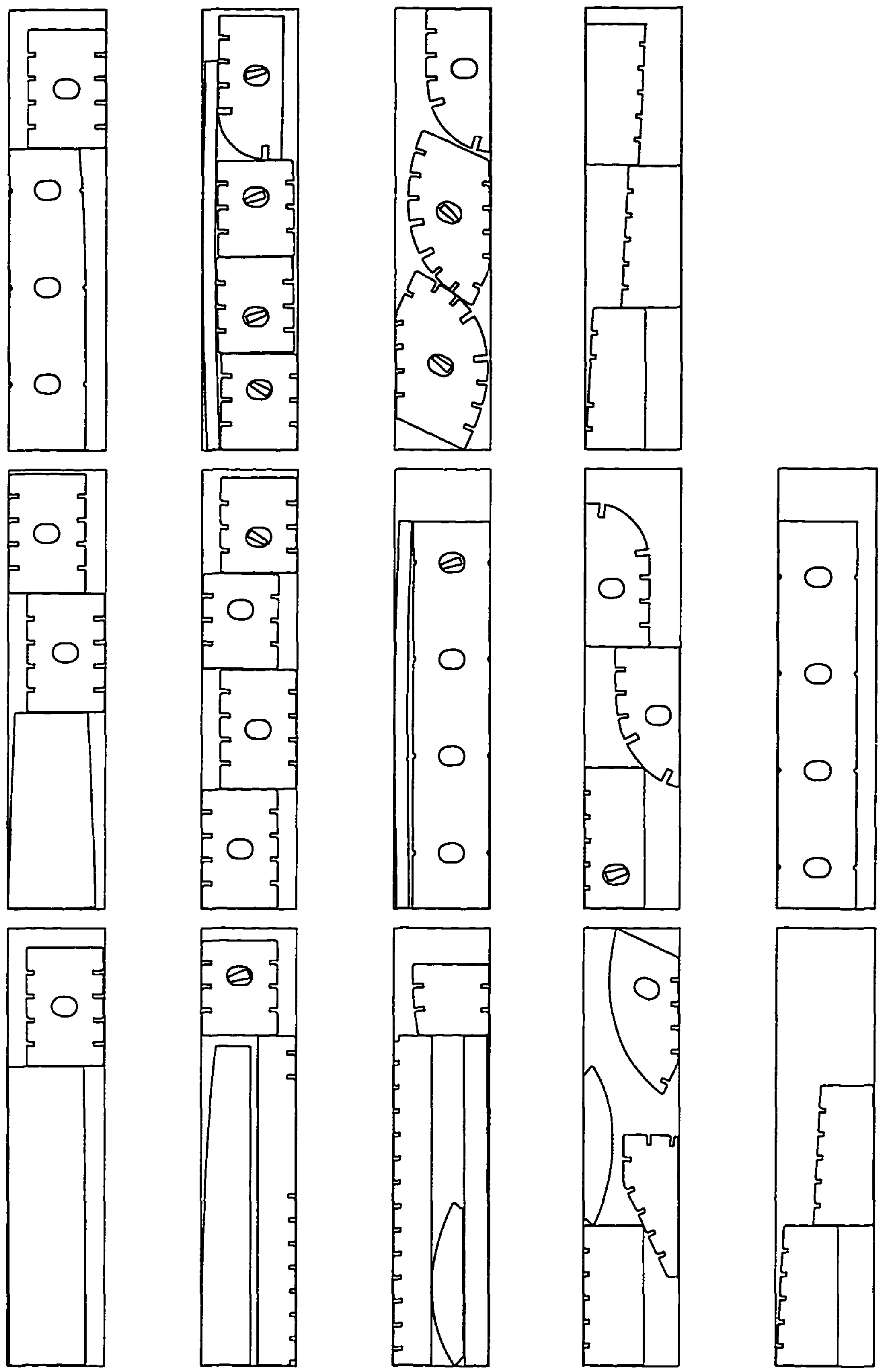


Figure A.3.2 Problem B

(fig. con'd.)



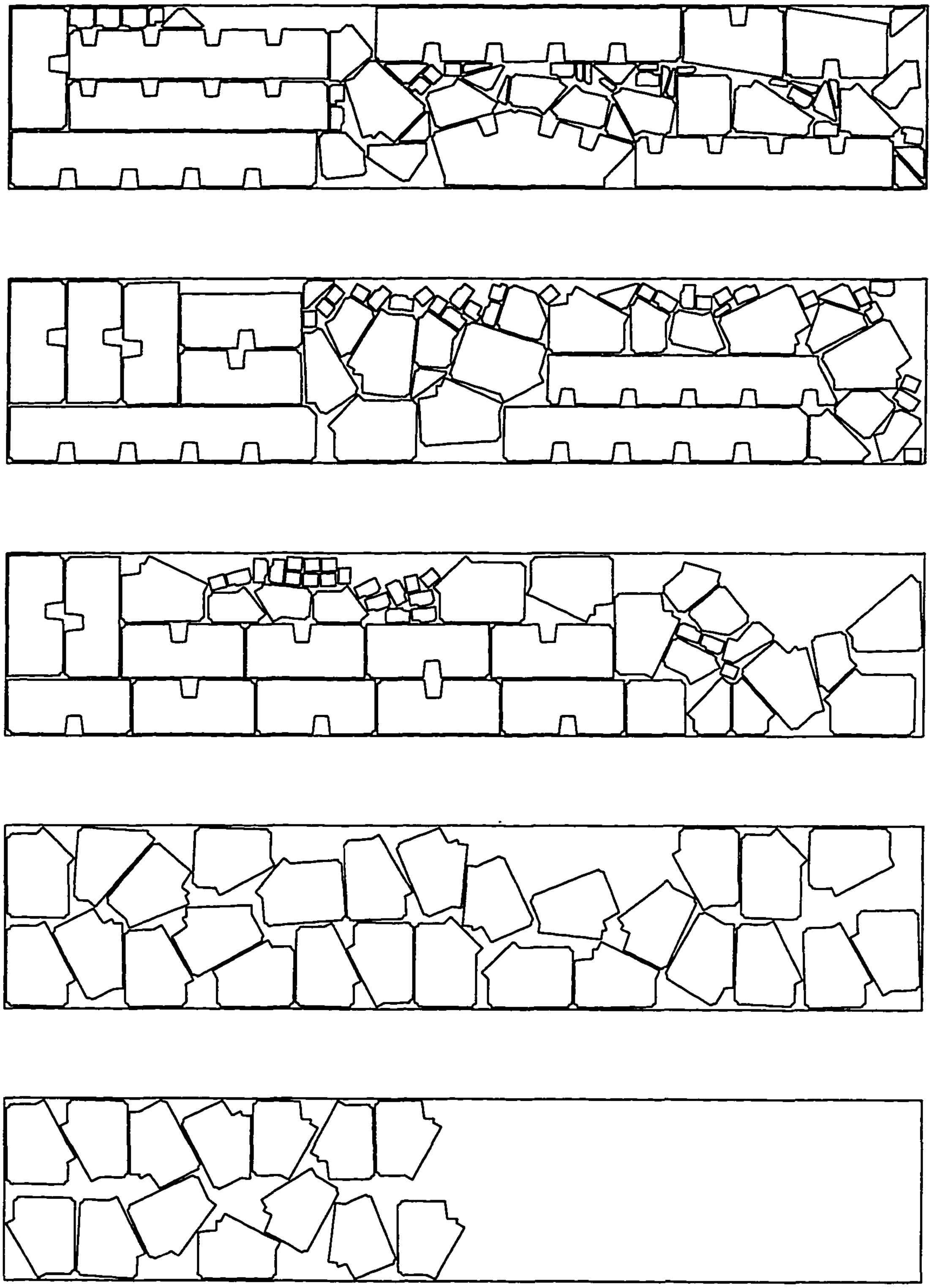


Figure A.3.3 Problem C

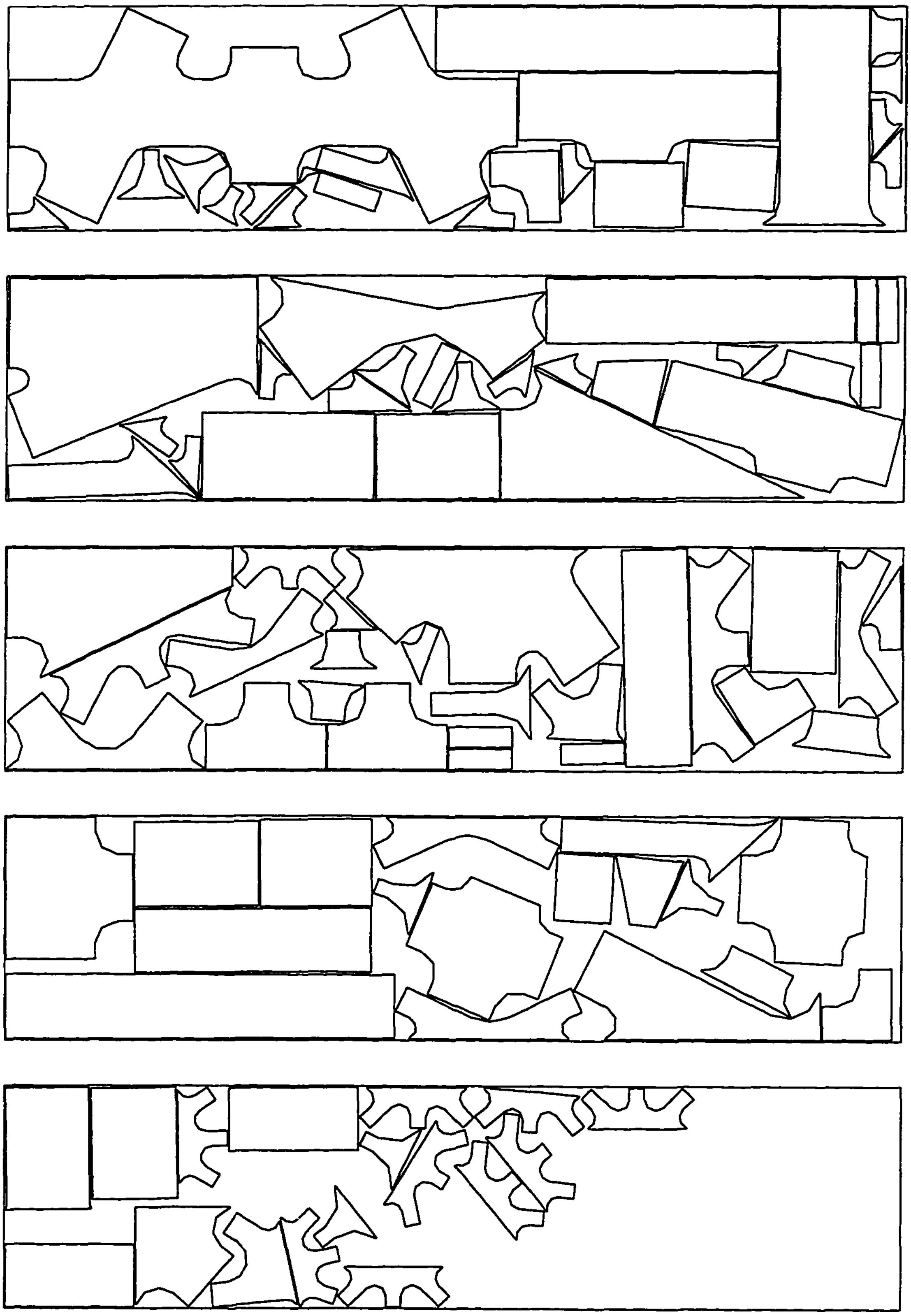


Figure A.3.4 Problem D

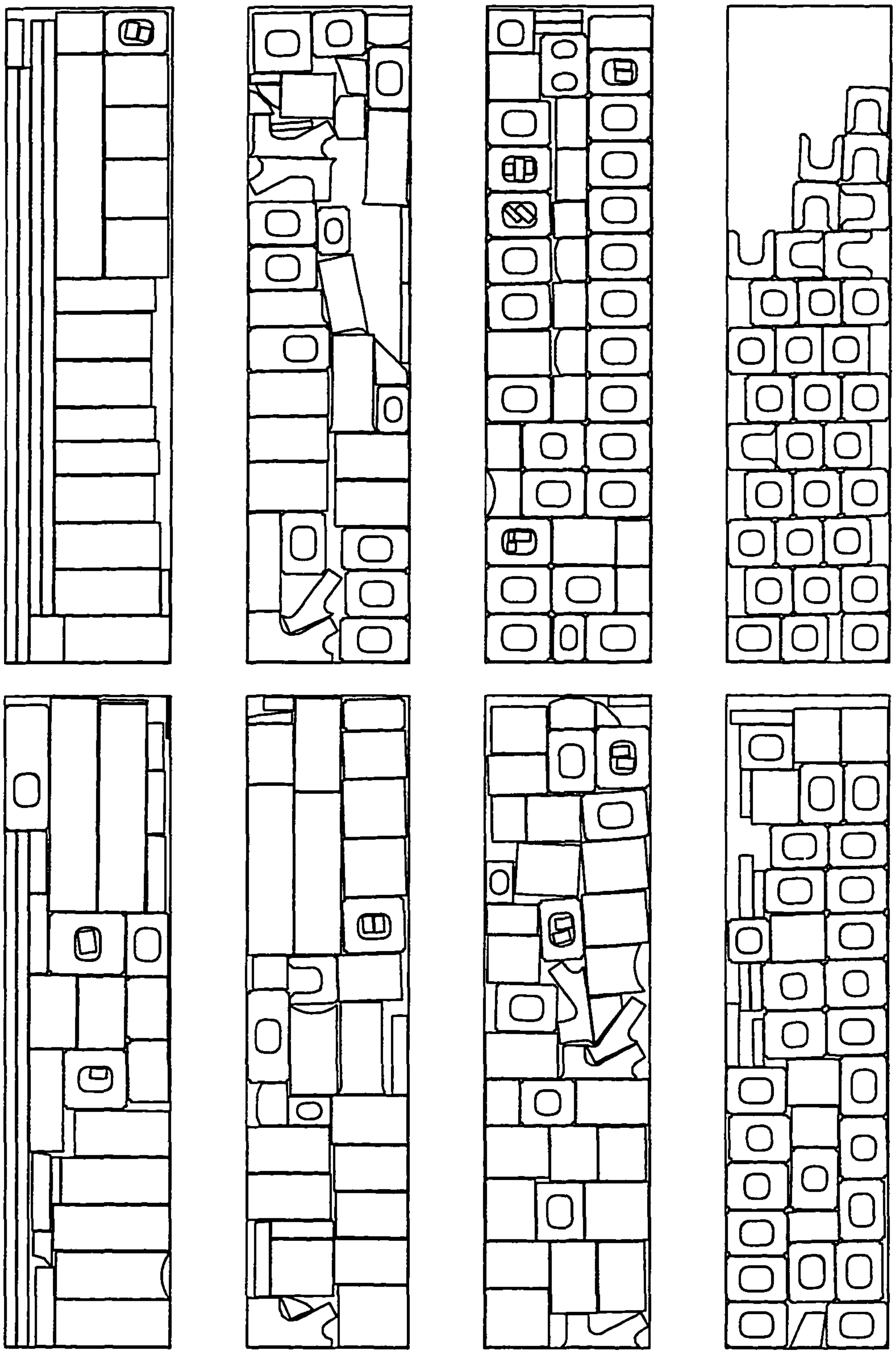


Figure A.3.5 Problem E

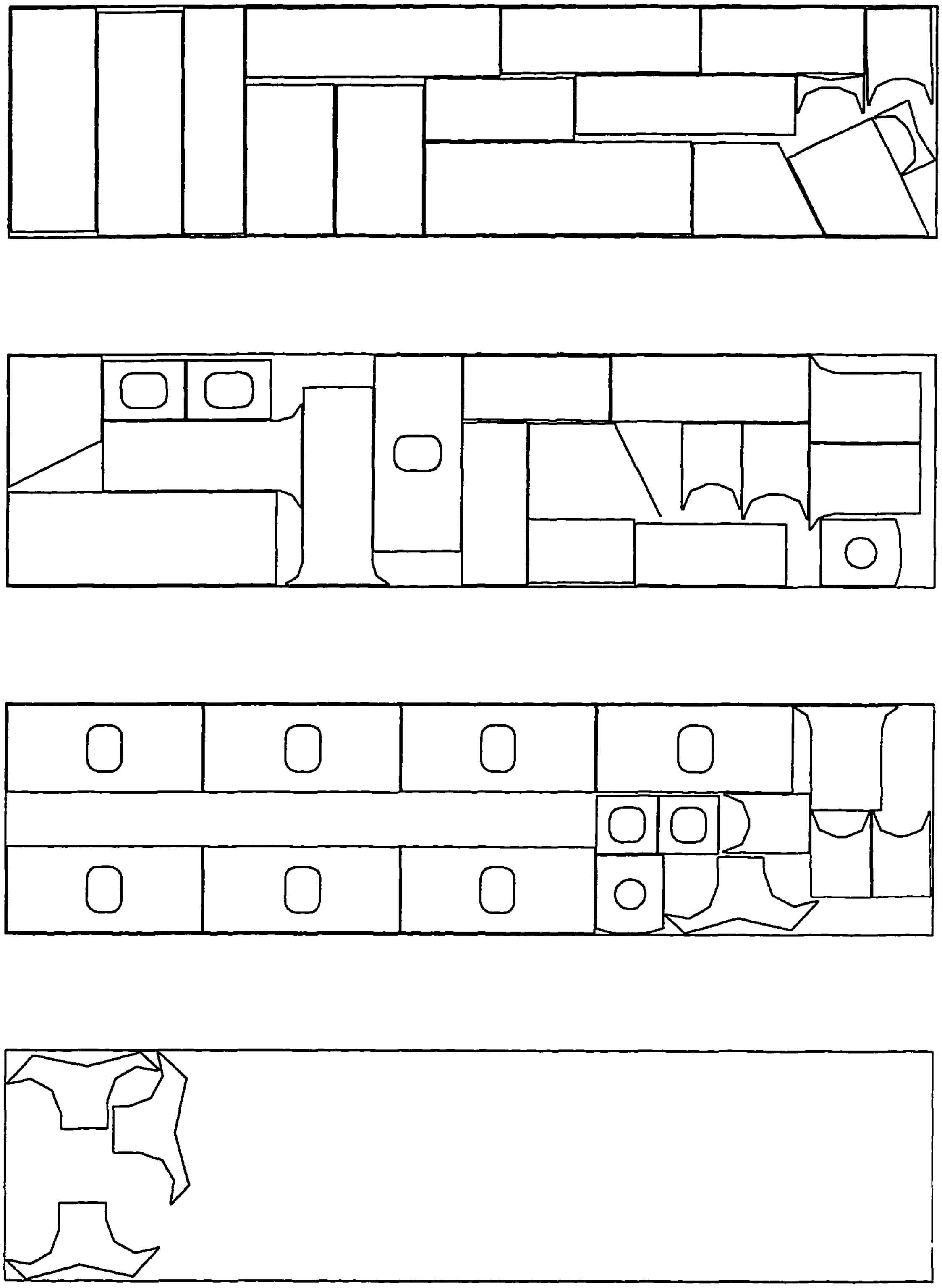


Figure A.3.6 Problem F

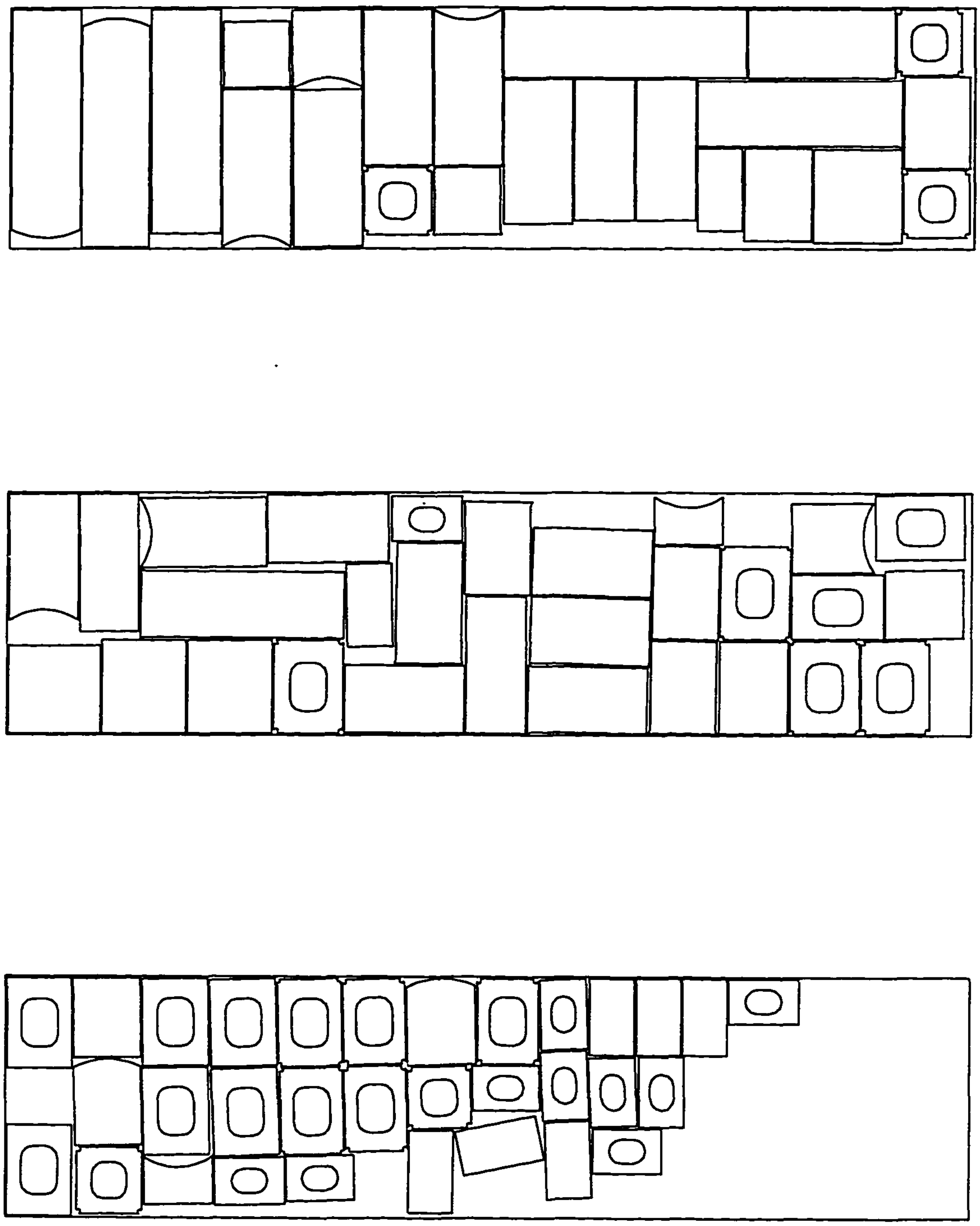


Figure A.3.7 Problem G

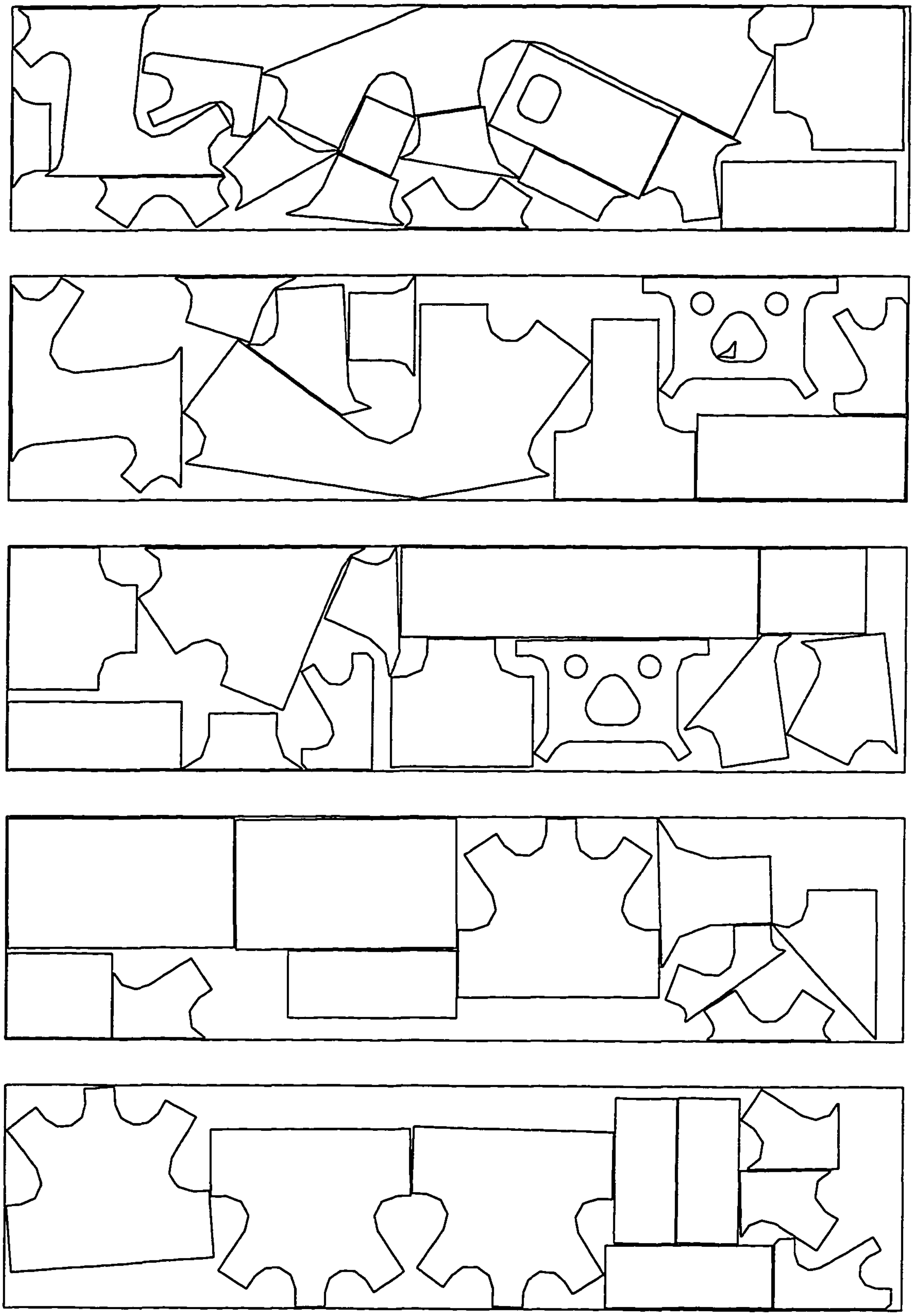


Figure A.3.8 Problem H

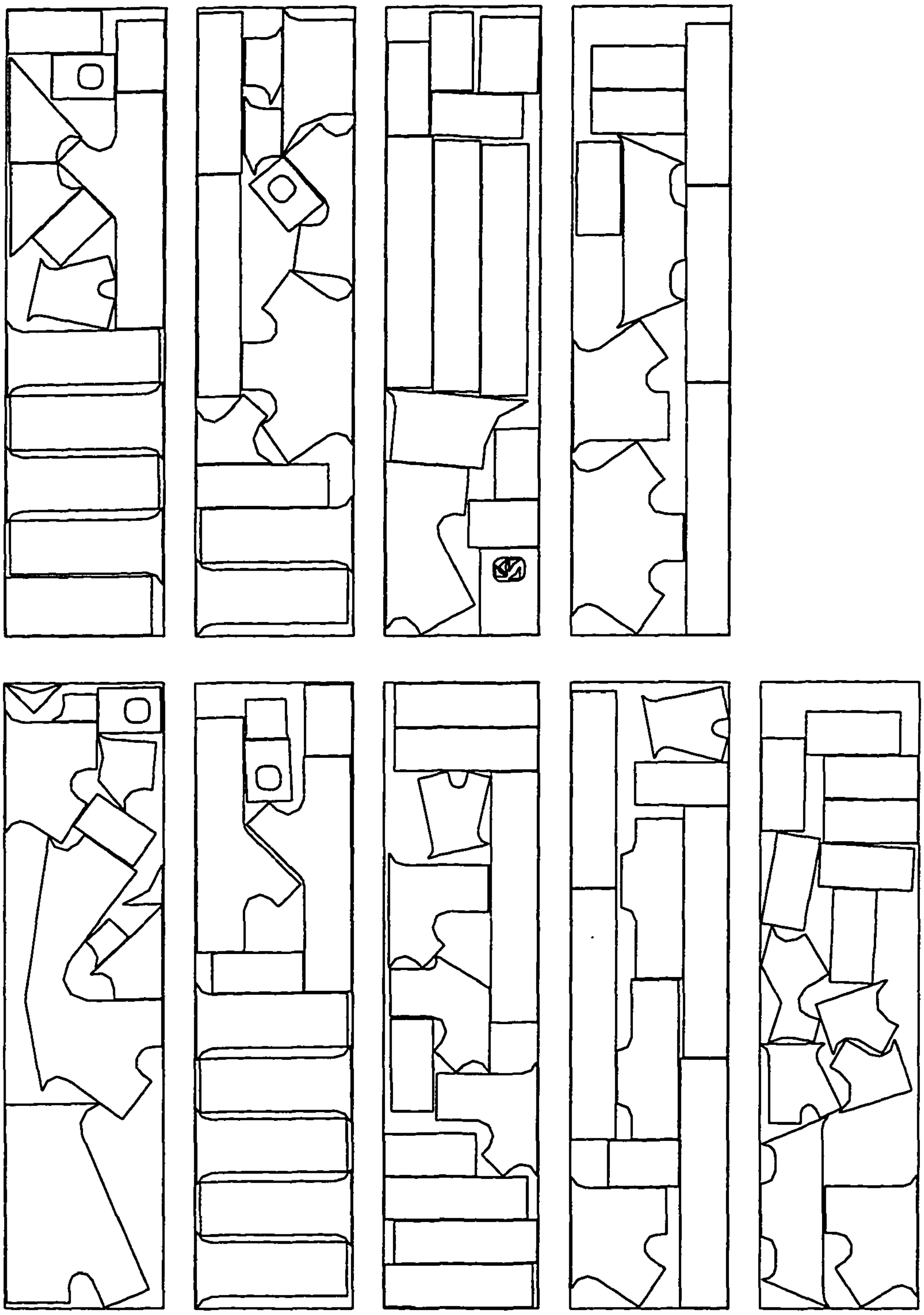


Figure A.3.9 Problem I

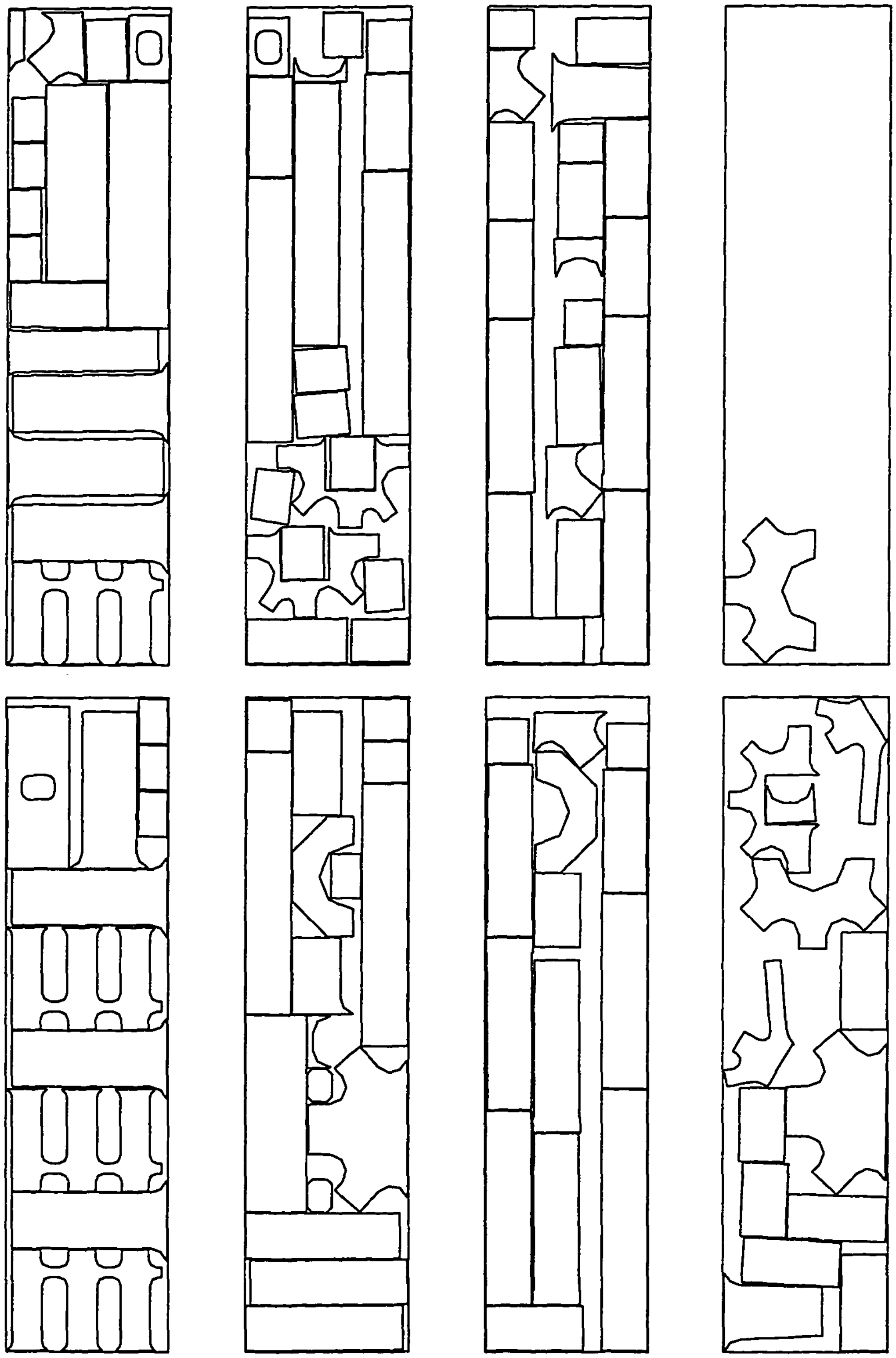


Figure A.3.10 Problem J

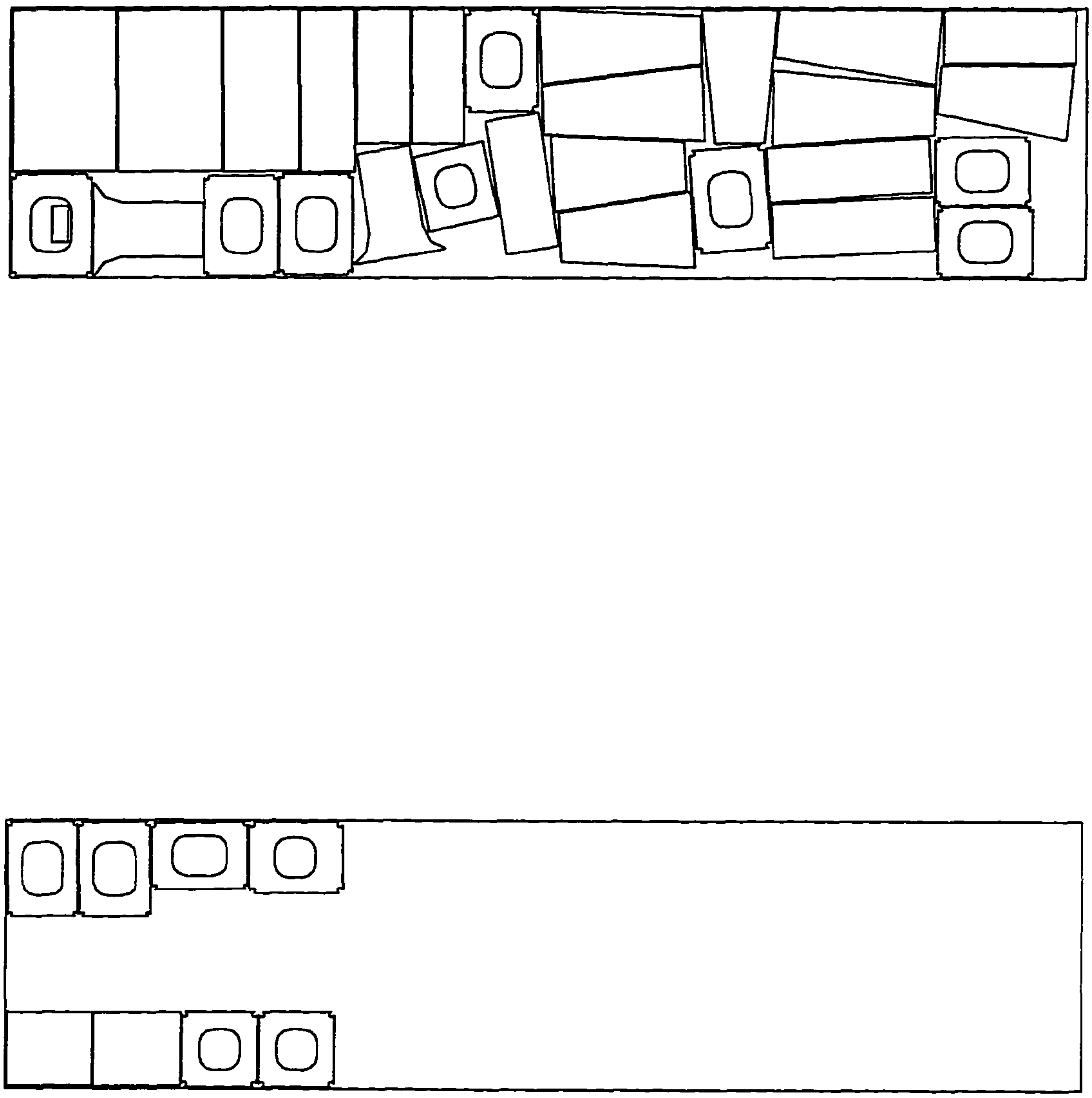


Figure A.3.11 Problem K

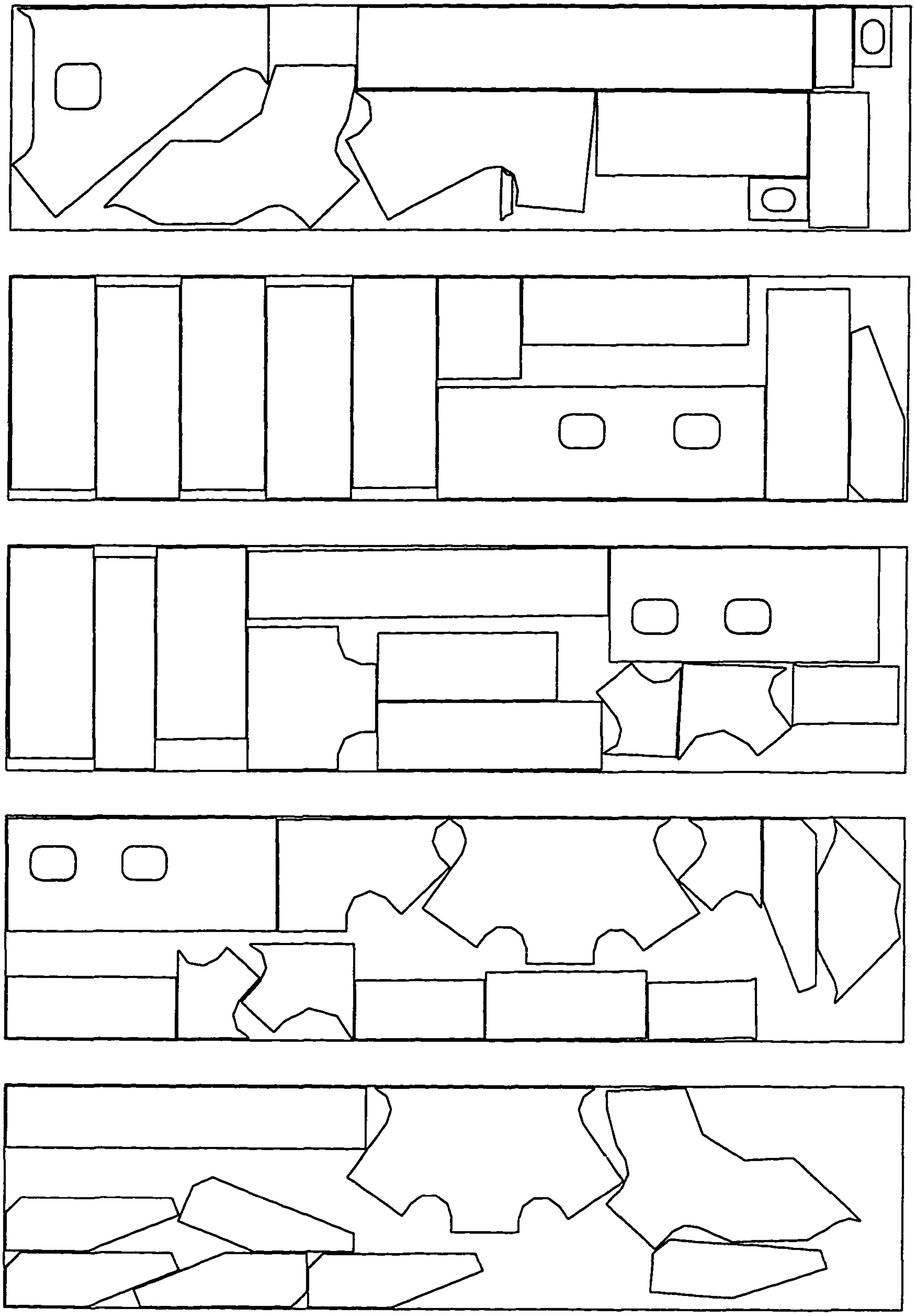


Figure A.3.12 Problem L

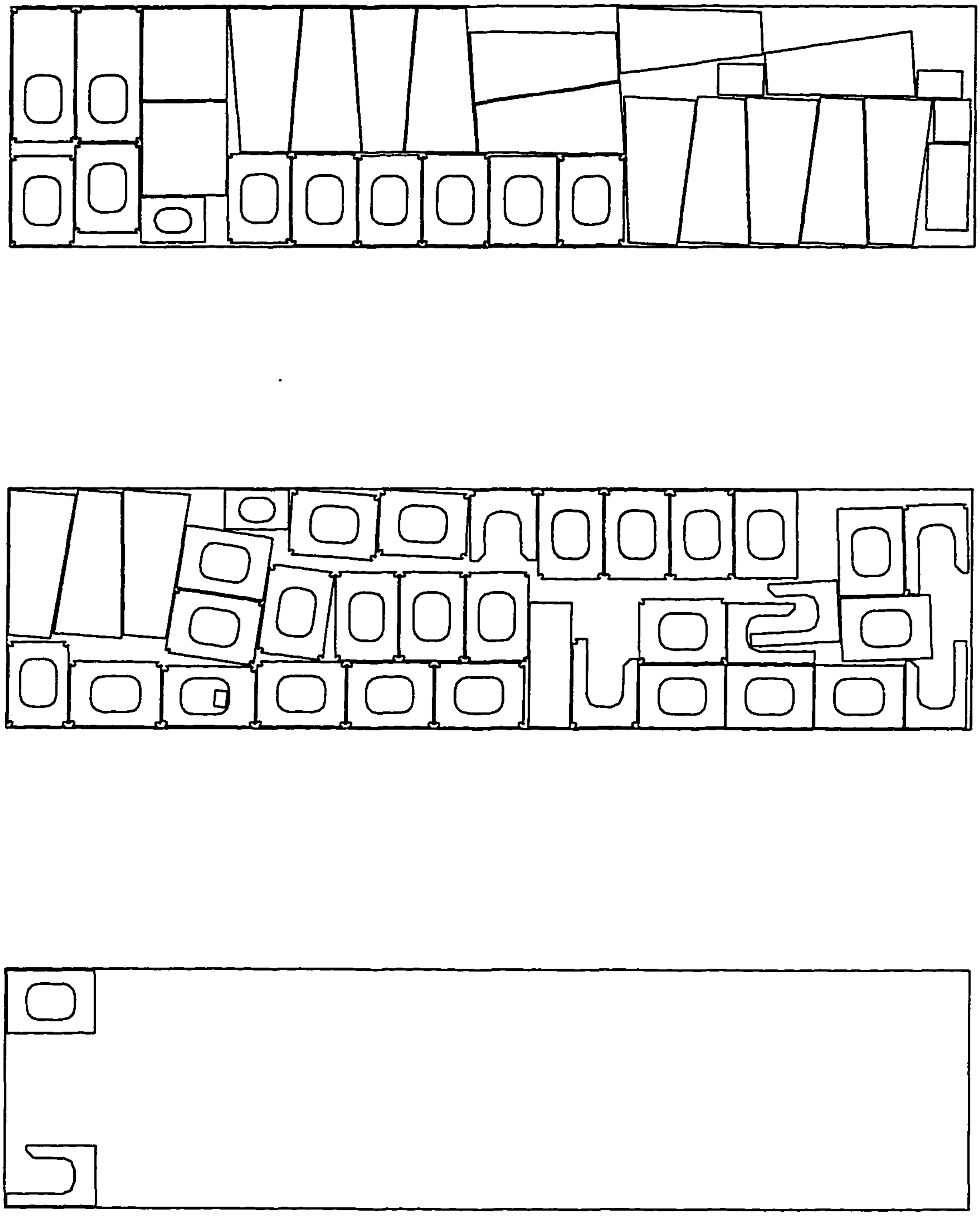


Figure A.3.13 Problem M

A.4 NFP Based Solution Layouts

Figures A.4.1 through A.4.13 show NFP based solutions for the problems of the parameter study. Layouts are plotted in the order of their creation from left to right across each row. Backtracking is disabled for all runs by setting the bandwidth to zero. Additional information for each problem concerning the profile characteristics, CPU times, and trim waste can be found in Tables 5.2, 5.4, and 5.5.

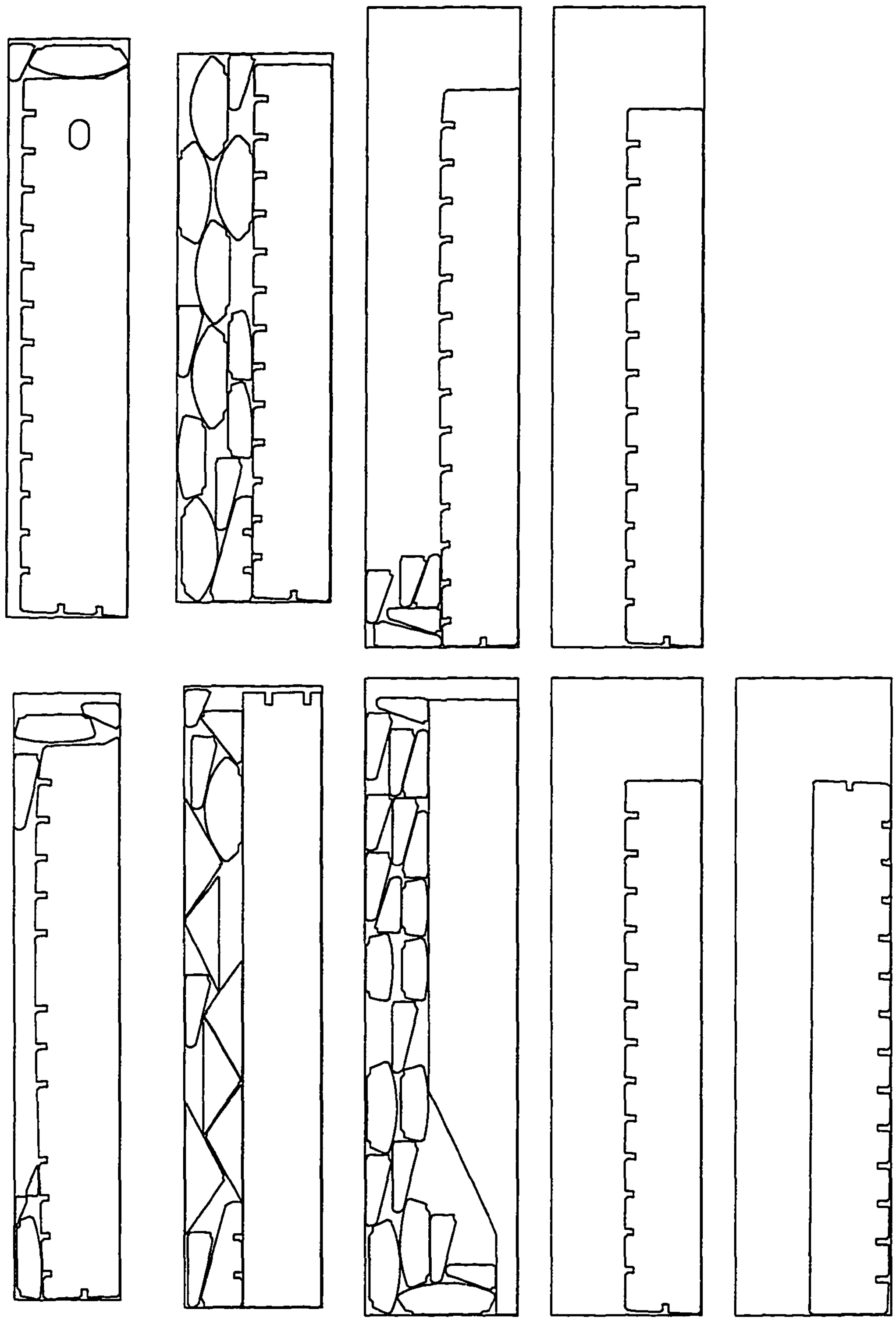


Figure A.4.1 Problem A

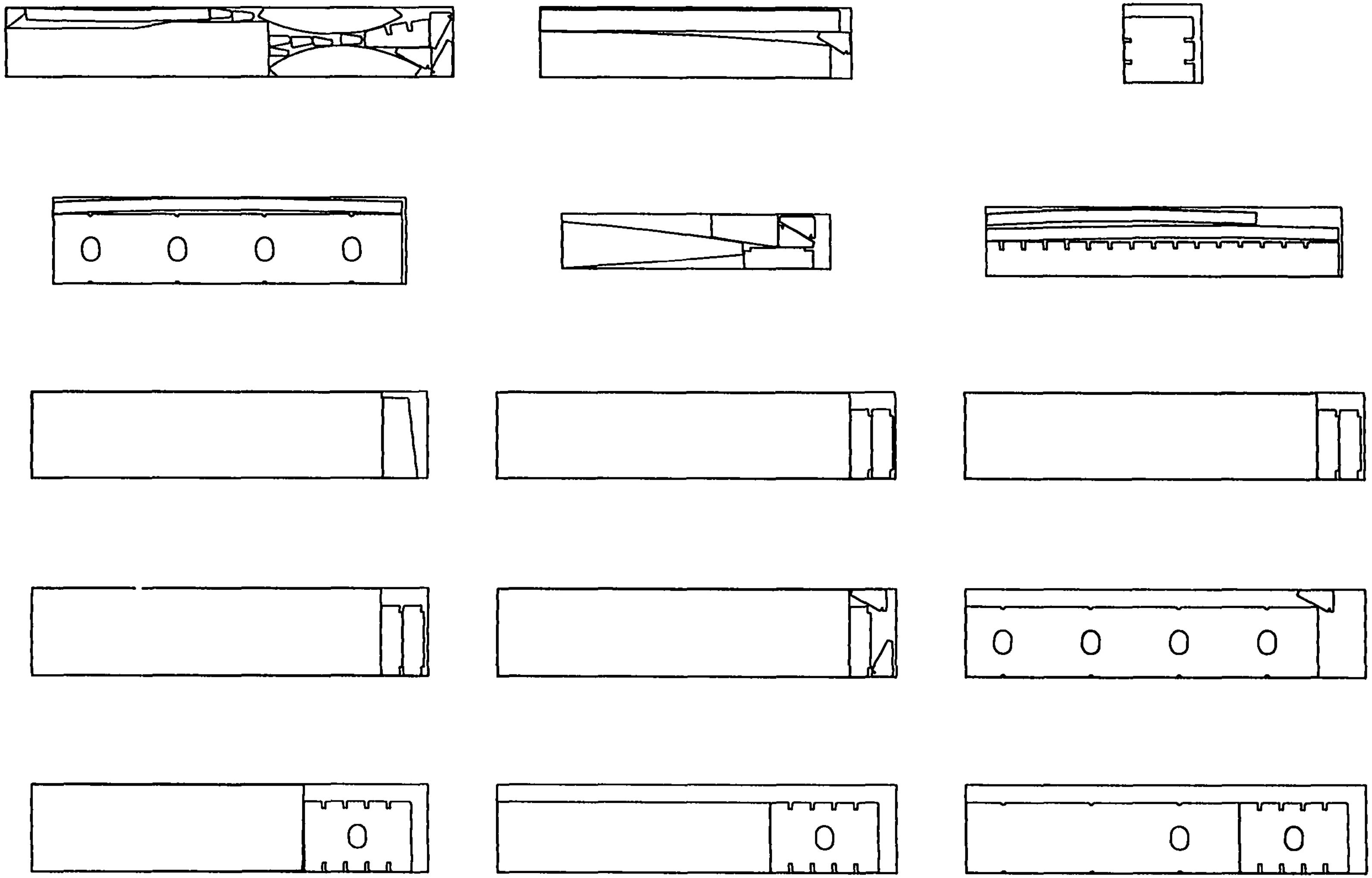
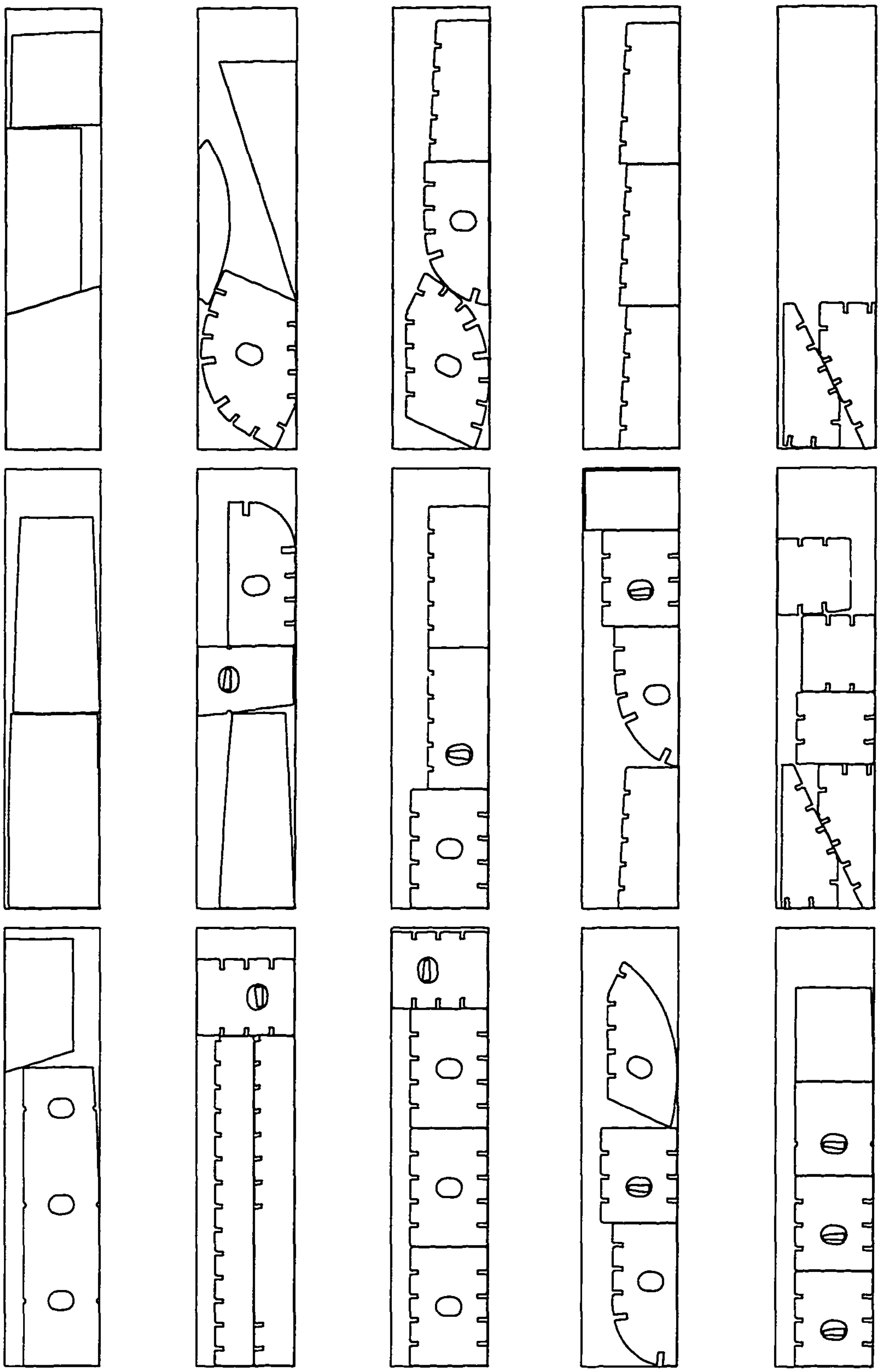


Figure A.4.2 Problem B

(fig. con'd.)



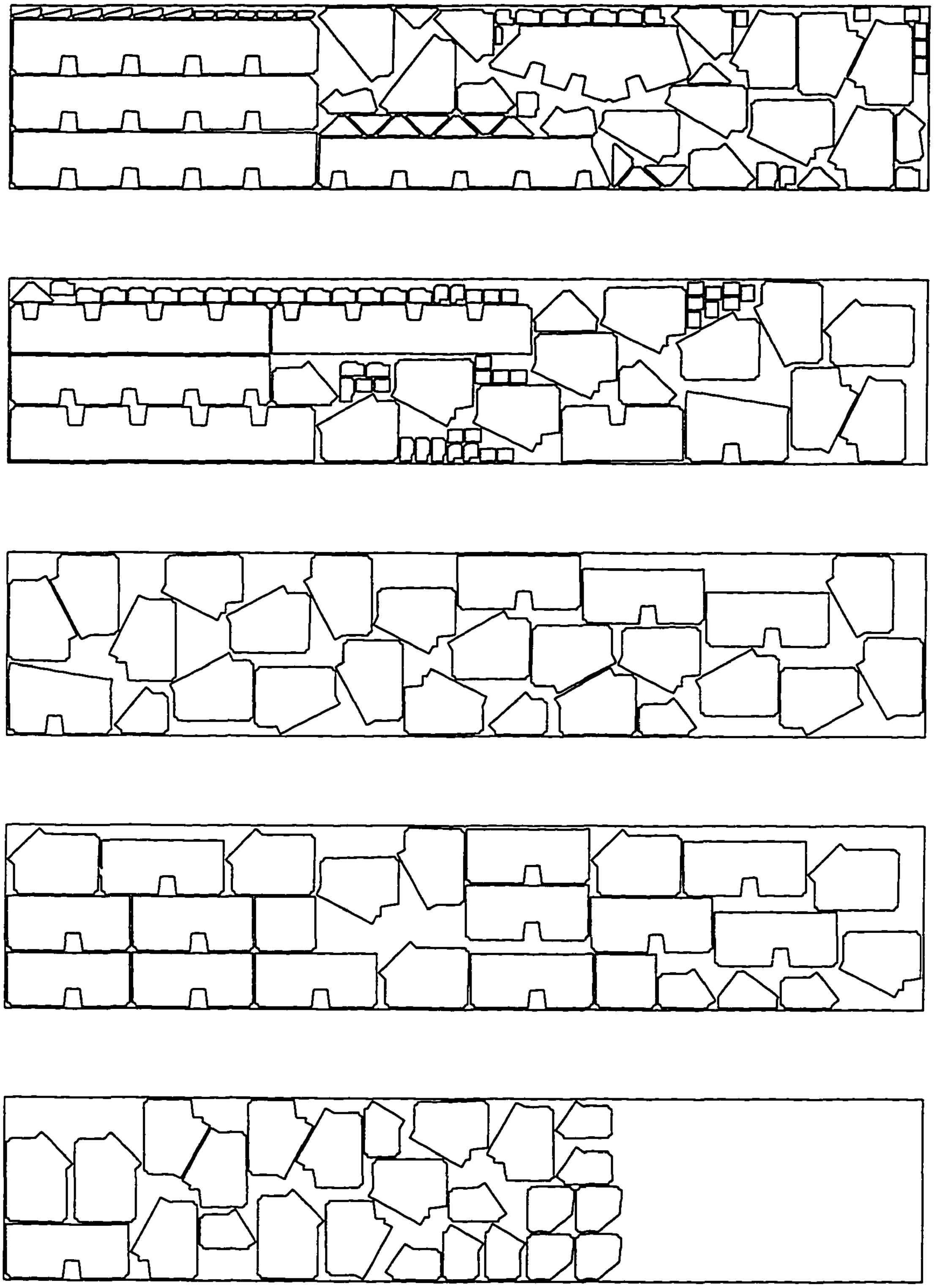


Figure A.4.3 Problem C

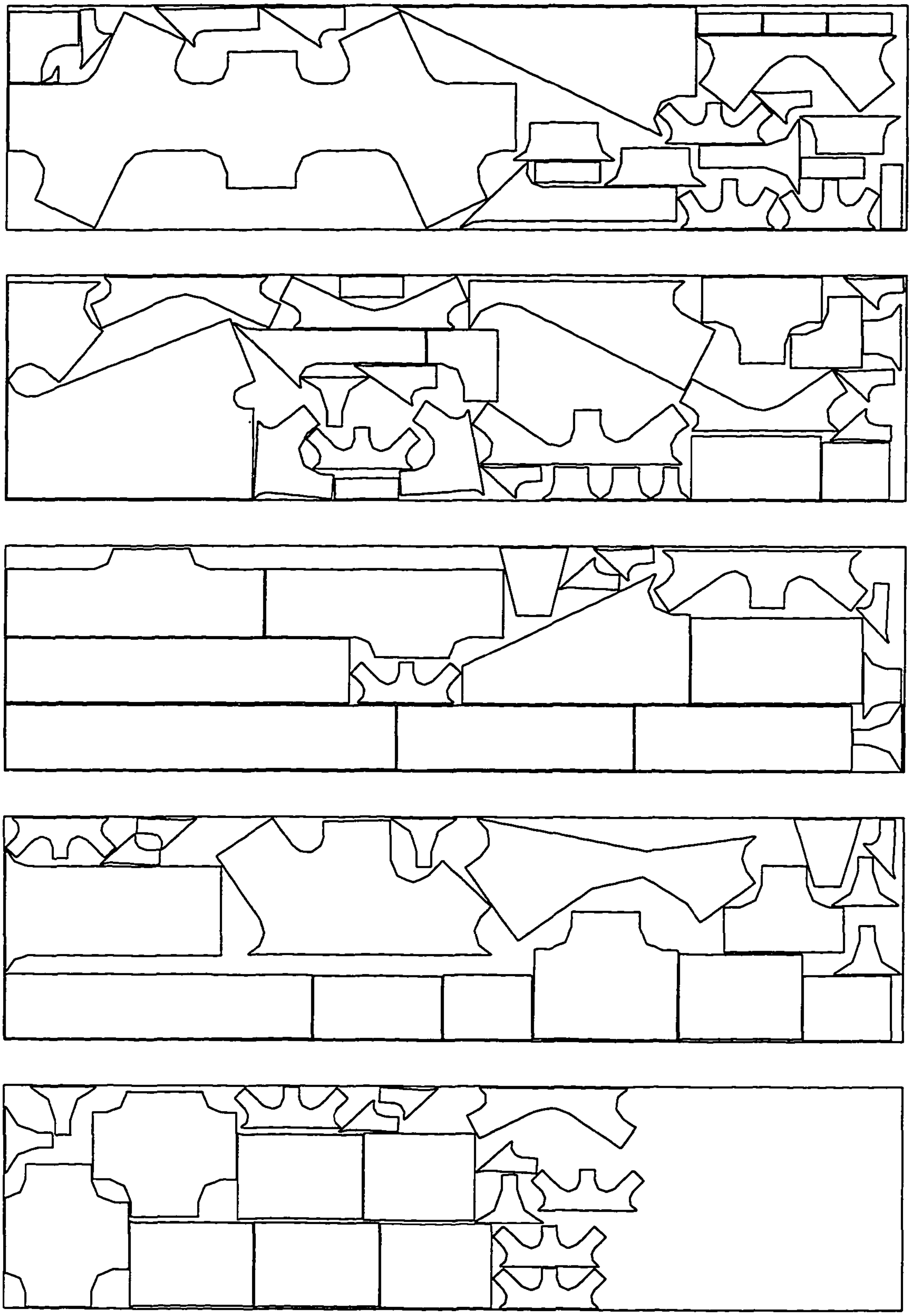


Figure A.4.4 Problem D

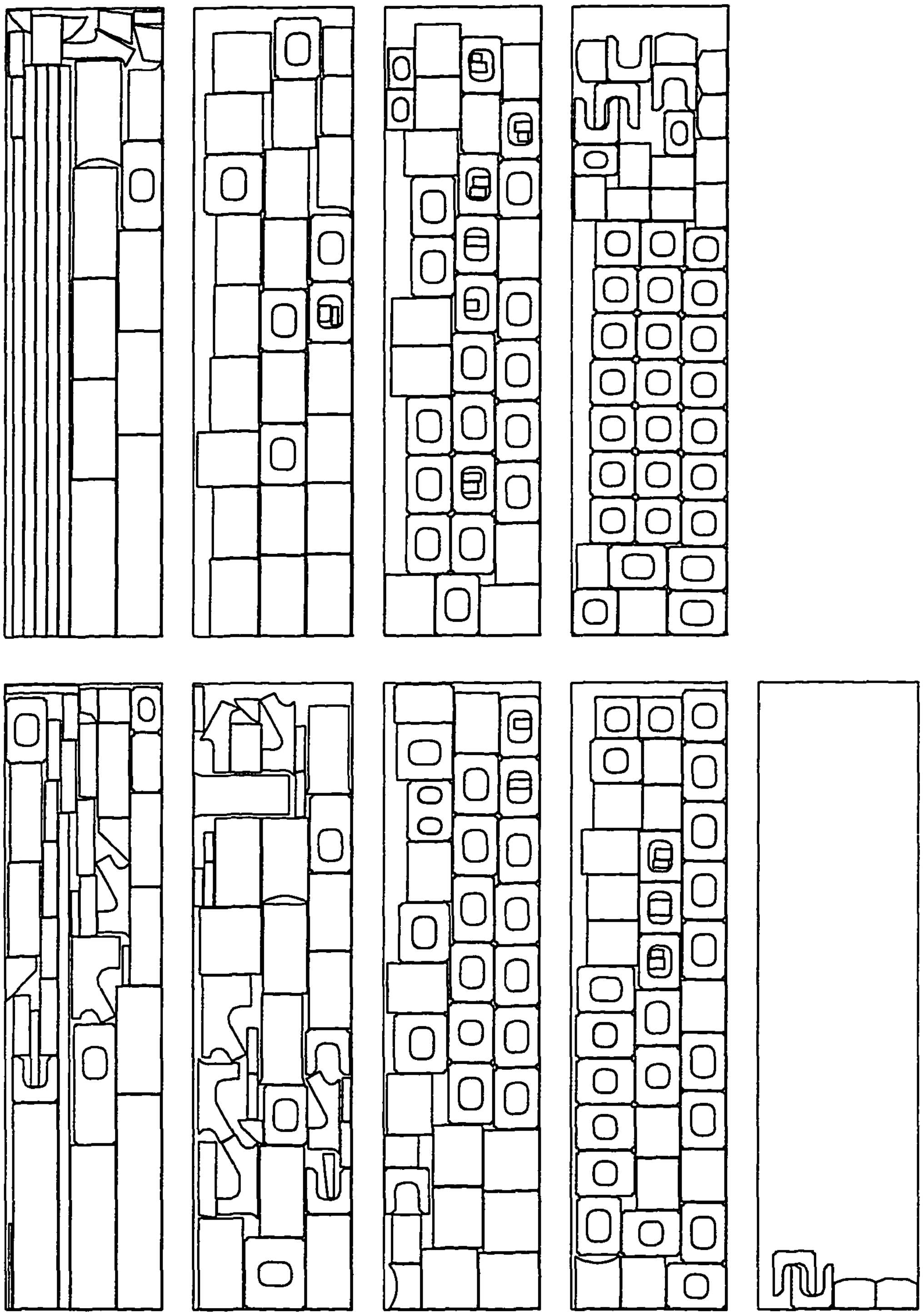


Figure A.4.5 Problem E

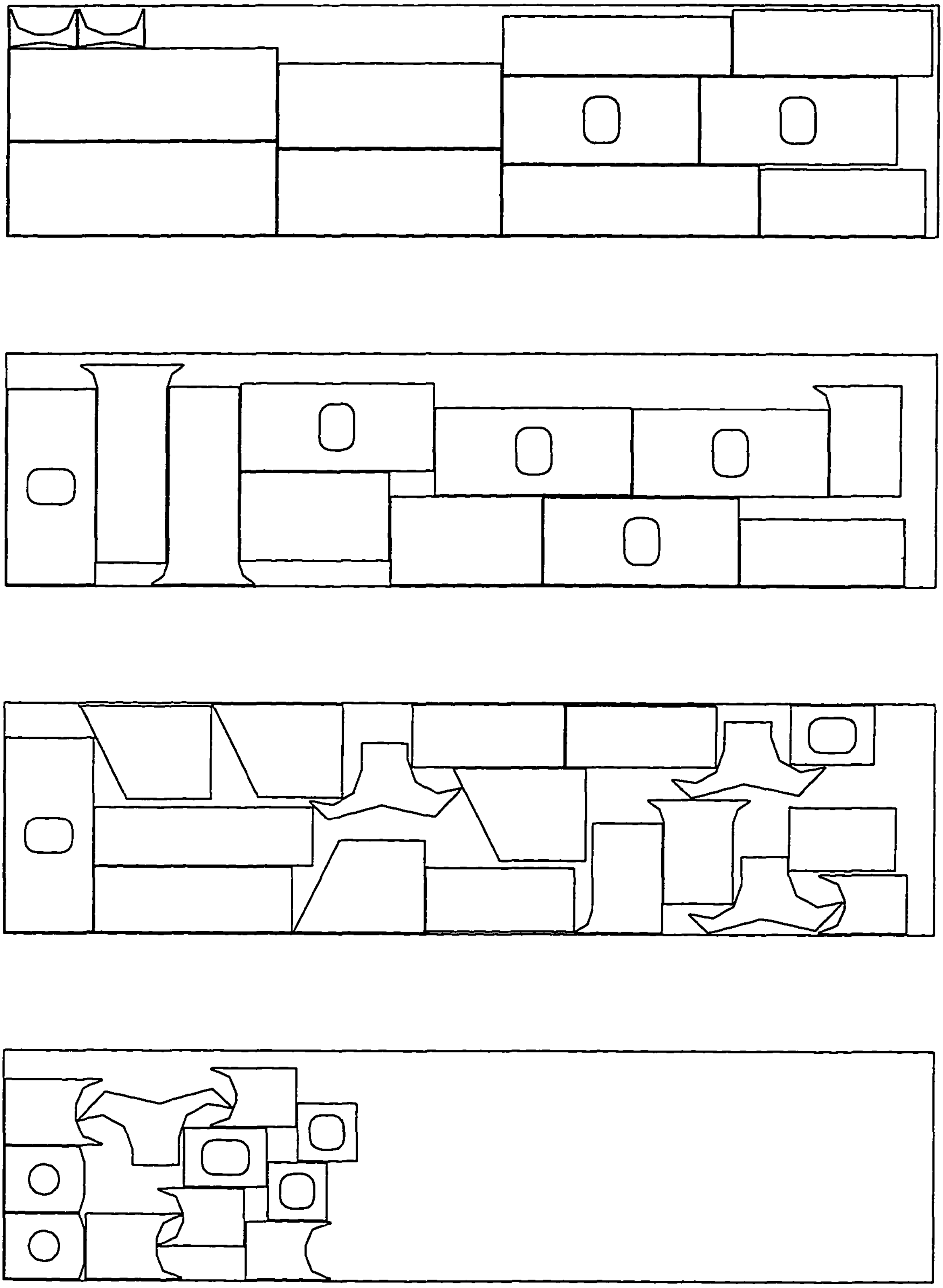


Figure A.4.6 Problem F

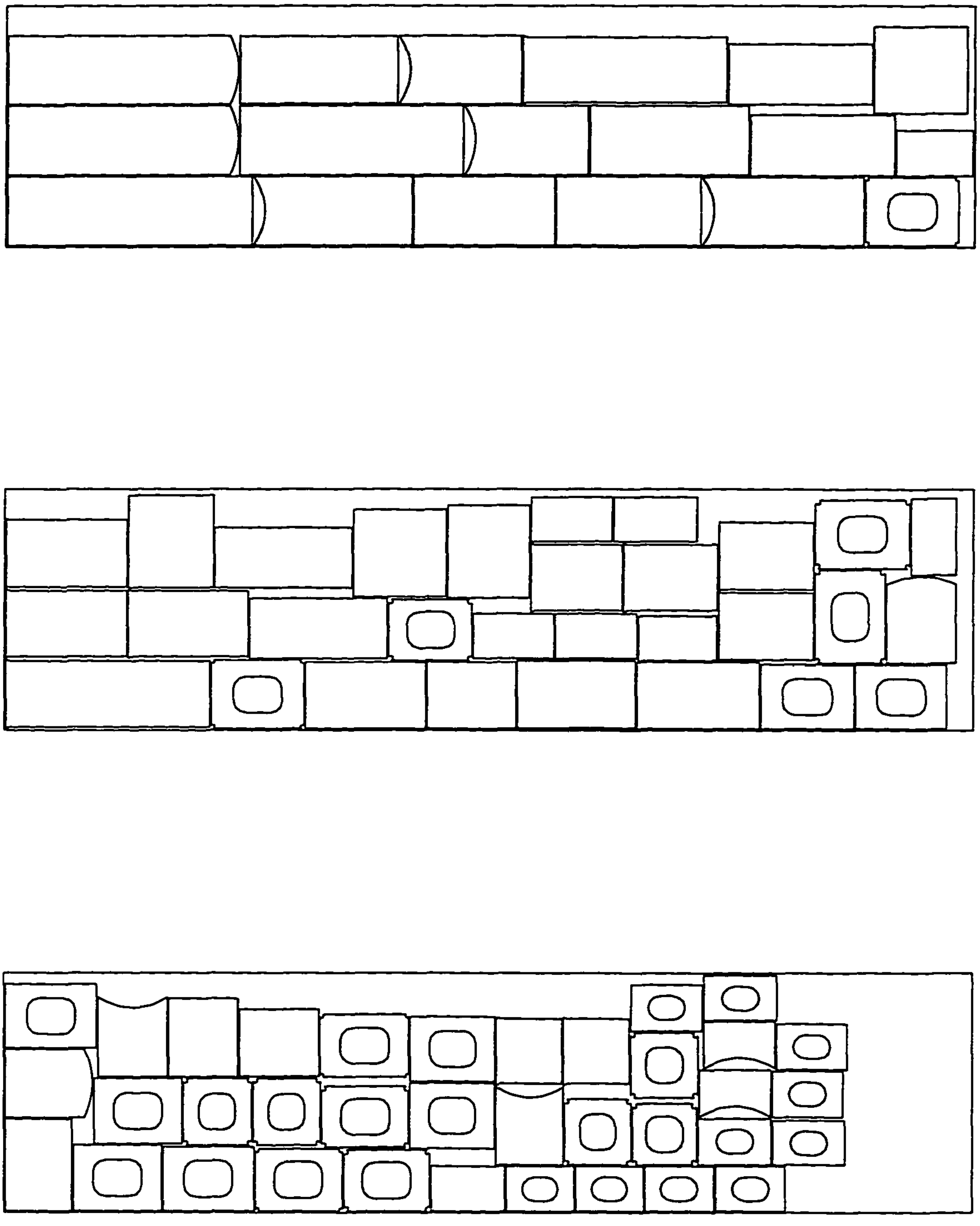


Figure A.4.7 Problem G

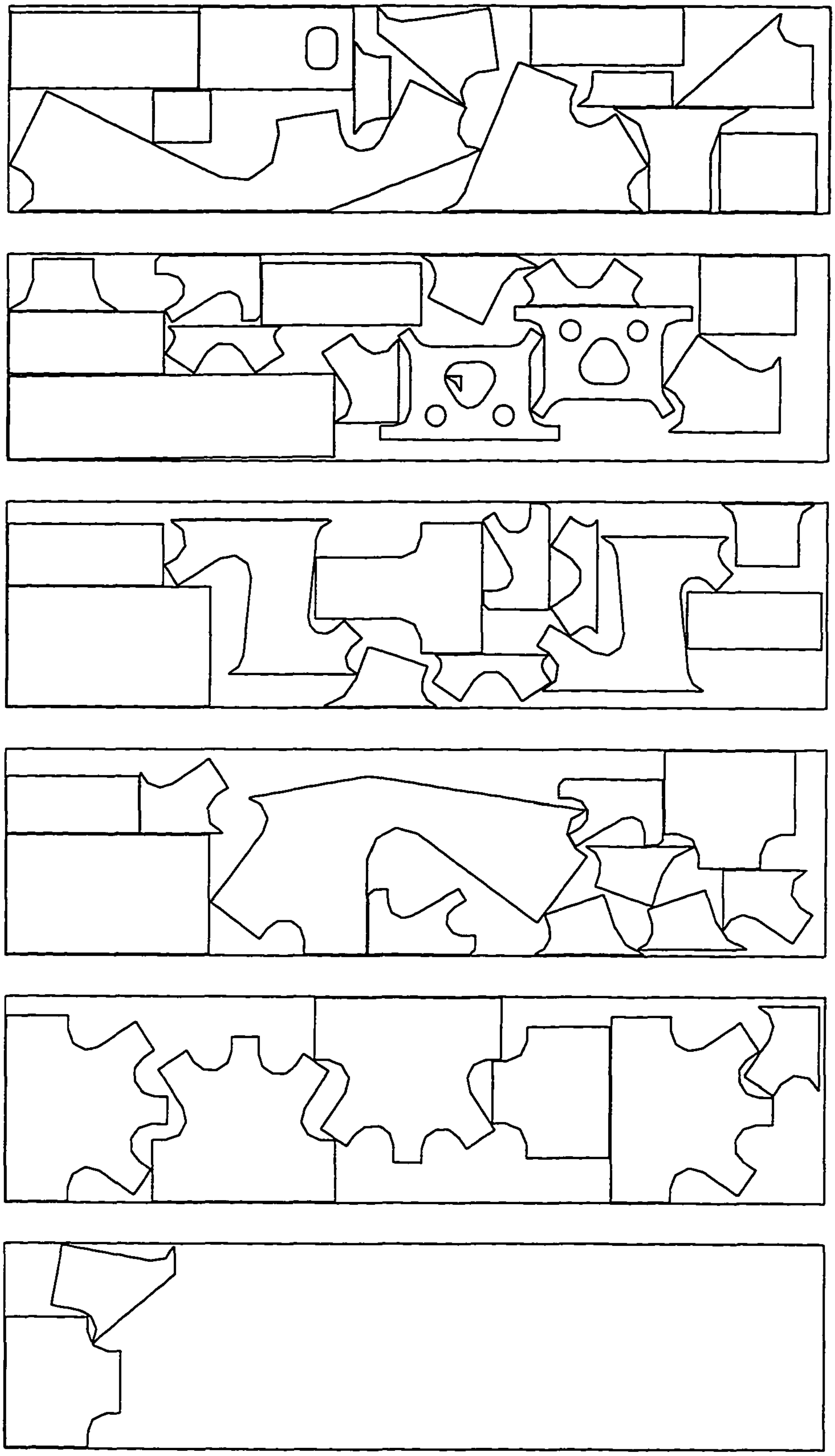


Figure A.4.8 Problem H

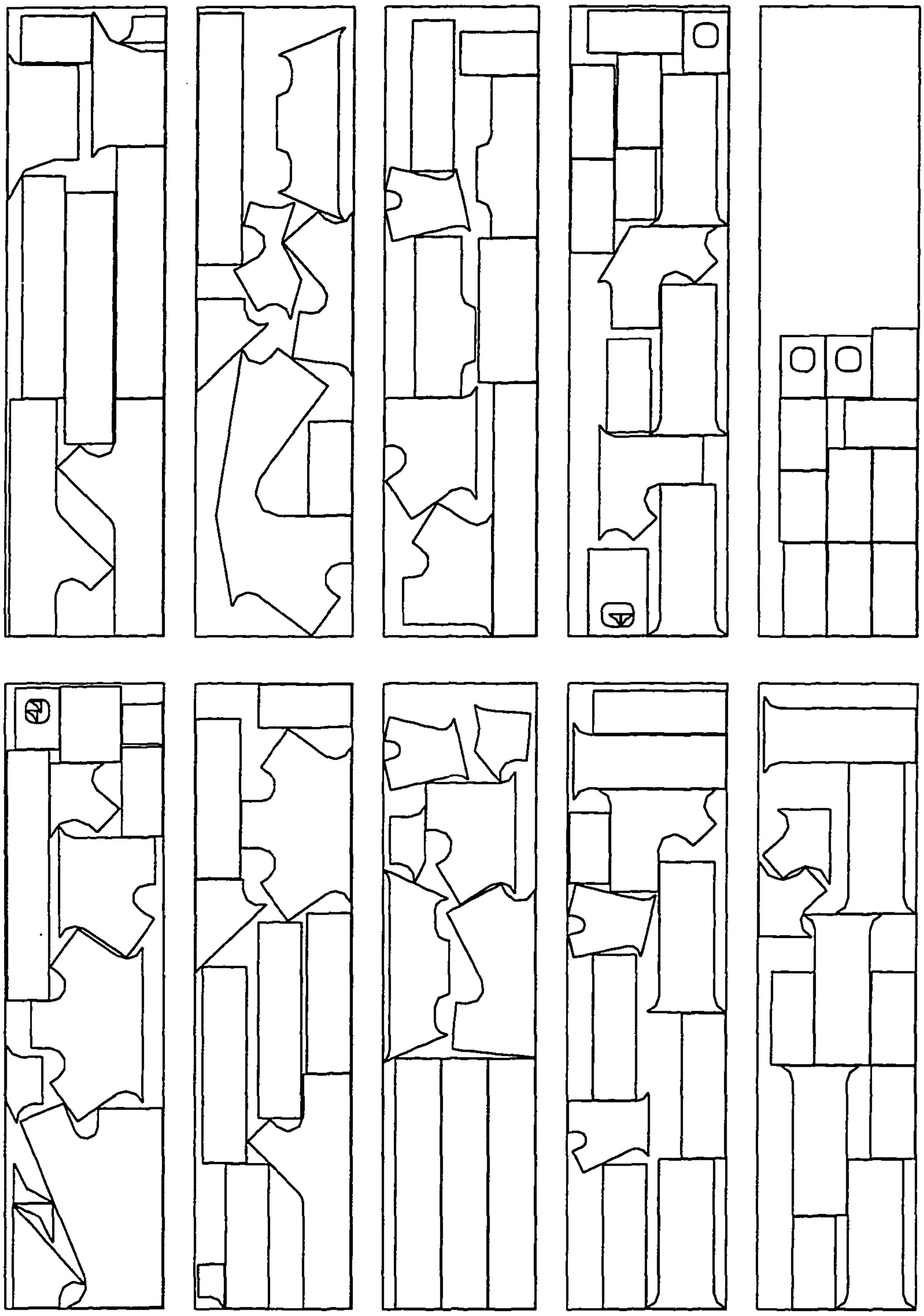


Figure A.4.9 Problem I

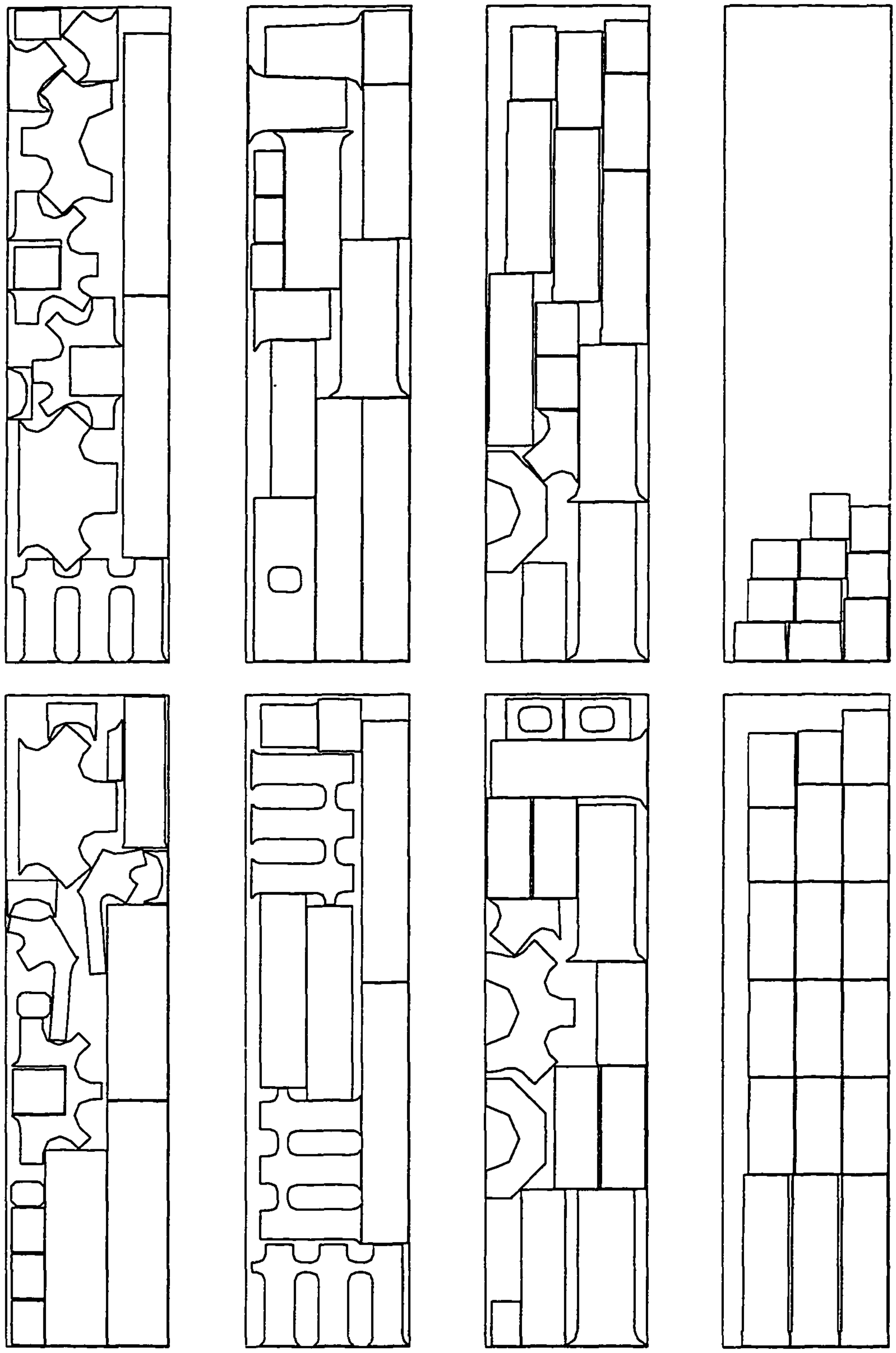
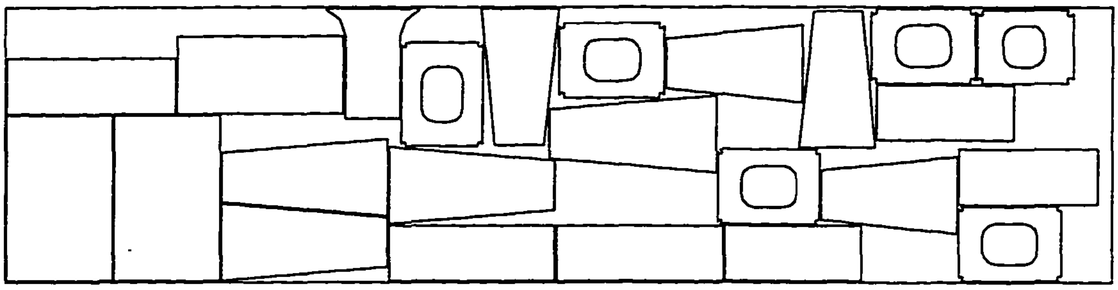


Figure A.4.10 Problem J



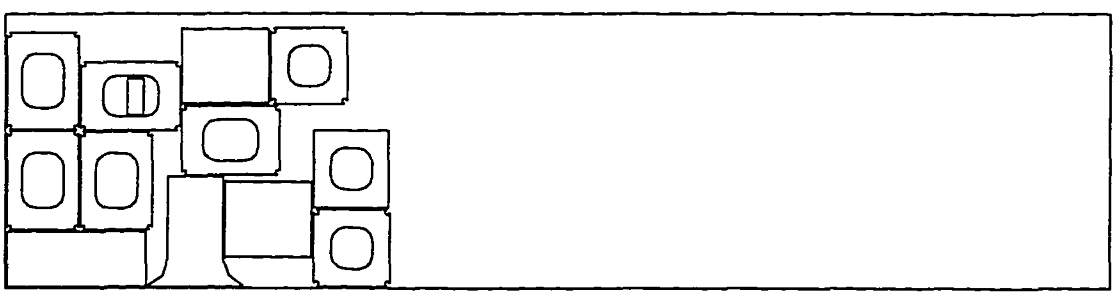


Figure A.4.11 Problem K

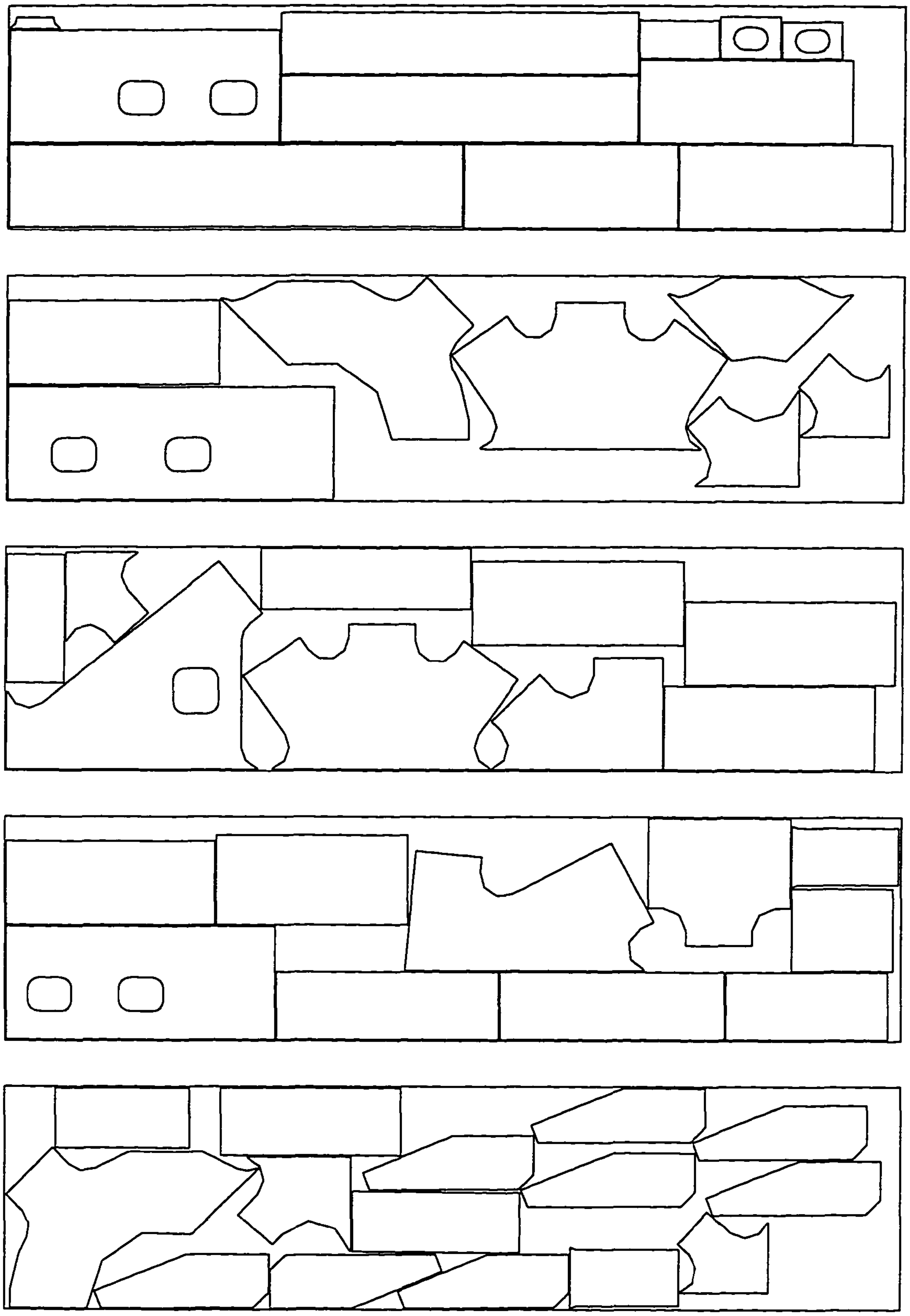


Figure A.4.12 Problem L

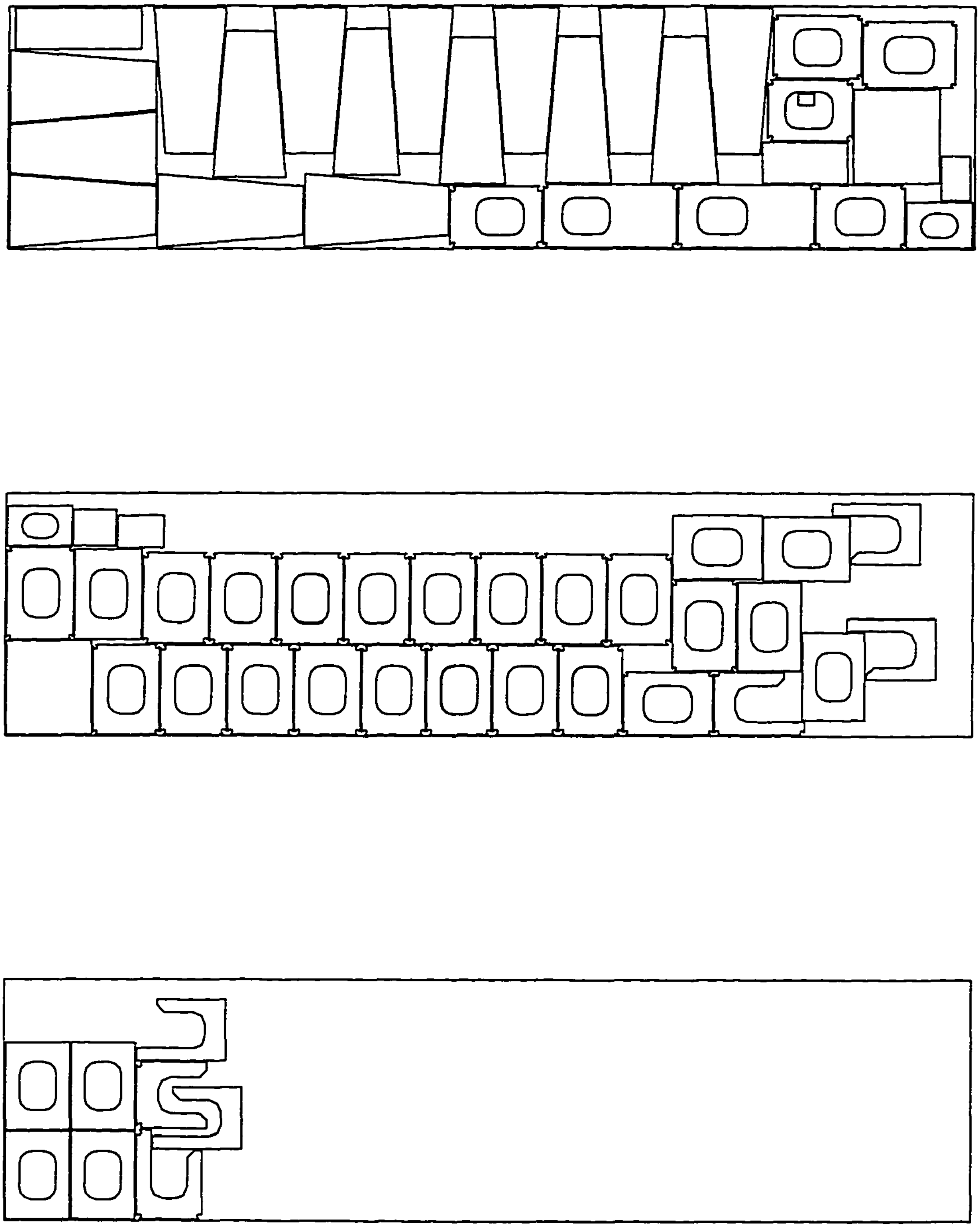


Figure A.4.13 Problem M

Vita

Henry was born on June 6, 1965 in New Orleans, Louisiana, to Rosemary and Henry Lamousin. He attended Brother Martin High School and graduated as the 1983 class valedictorian. Henry was awarded a Bachelors of Science in Mechanical Engineering from the University of New Orleans in 1987. After serving briefly as a project engineer at the U.S. Gypsum Company in Jacksonville Florida, he returned to Louisiana in 1989 to pursue a graduate degree. While at Louisiana State University he was the recipient of a Dean's Board of Regents Fellowship. His areas of concentration were design and computer aided geometric modeling. Henry completed his doctoral research in September 1996.

DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: Henry J. Lamousin III

Major Field: Mechanical Engineering

Title of Dissertation: Allocation of Two Dimensional Parts Using a Shape Reasoning Heuristic

Approved:

Warren N. Waggenspack Jr.
Major Professor and Chairman

John M. Larkin
Dean of the Graduate School

EXAMINING COMMITTEE:

Neal W. Stoltzfus

Michael C. Murphy

Dirk E. Smith

S. Iyengar

W. Jiao

Date of Examination:

September 12, 1996